8.NS.1 Answers

1. A
2. C
3. C
4. C
5. C, E, F
6. A, B, E
7. E
8. F
9. D
10. B
11. A
12. a. Alex answered $\frac{21}{25} = 0.84$ of the questions correctly.

    b. 80% of 25 is $0.80 \cdot 25 = 20$, so 20 questions need to be answered correctly.

    c. Yes. Alex scored at least 80% on his final exam. Reason 1: He answered $0.84 = 84\%$ of the questions correctly, and $84\% > 80\%$. Reason 2: He answered 21 questions correctly, and $21 > 20$.

    Rubric

    a. 1 point
    b. 1 point
    c. 2 points

13. A rational number has a decimal expansion that either terminates or repeats. The three dots at the end of the given decimal indicate that the decimal does not terminate. Although there is a pattern to the digits in the decimal (the number of 1s following each 0 after the decimal point increases by 1 as you examine the digits from left to right), there is no block of digits that gets repeated, so the decimal does not repeat. A nonterminating, nonrepeating decimal represents an irrational number.

    Rubric

    1 point for identifying the number as irrational; 1 point for a correct explanation

14. The number of students per computer is $\frac{333}{99} = \frac{37}{11} = 3.\overline{36}$.

    Rubric

    1 point for the correct simplified rational number; 1 point for the correct decimal

15.
$$x = 0.3\overline{4}$$
$$10x = 3.\overline{4}$$
$$10x - x = 3.\overline{4} - 0.3\overline{4}$$
$$9x = 3.1$$
$$x = \frac{3.1}{9} \cdot \frac{10}{10} = \frac{31}{90}$$

    Rubric

    1 point for the correct answer; 1 point for the correct method of obtaining the answer

16. The decimal form of $\frac{22}{7}$ is $3.\overline{142857}$, which agrees with the decimal form of π to two decimal places.

    Rubric

    1 point for the correct decimal form of $\frac{22}{7}$; 1 point for the correct number of decimal places of agreement between $\frac{22}{7}$ and π

17. a. Decimals terminate only in cases where the prime factorization of *b* contains powers of 2 and/or 5. In those cases, an equivalent fraction can be written with a power of 10 as its denominator, which means the decimal terminates. Denominators like 2, 4, 5, 8, 20, and 25 will result in terminating decimals because the prime factorizations of these numbers include only powers of 2 and/or 5.

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17. b. The repeating block can have at most $(b-1)$ digits, because the remainder at each step of performing long division must be less than b. Therefore, the only possible remainders are 1, 2, …, $b-1$, and once they're used up, the remainders will have to start repeating, which means that the digits in the quotient will start repeating.

Rubric
2 points for each part

18. The decimal equivalent of $\frac{1}{11}$ is $0.\overline{09}$.

The decimal equivalent of $\frac{2}{11}$ is $0.\overline{18}$.

The decimal equivalent of $\frac{3}{11}$ is $0.\overline{27}$.

Since $0.\overline{09} = 1(0.\overline{09})$, $0.\overline{18} = 2(0.\overline{09})$, and $0.\overline{27} = 3(0.\overline{09})$, the decimal equivalent of $\frac{9}{11}$ must be $0.\overline{81}$ because $9(0.\overline{09}) = 0.\overline{81}$.

Rubric
1 point for the correct decimal equivalent of $\frac{1}{11}$; 1 point for the correct decimal equivalent of $\frac{2}{11}$; 1 point for the correct decimal equivalent of $\frac{3}{11}$; 1 point for a correct explanation of how to predict the decimal equivalent of $\frac{9}{11}$

19. a. $\frac{22}{25} = \frac{88}{100} = 0.88$

The decimal equivalent of $\frac{22}{25}$ is 0.88.

Since Marcos wrote that the decimal equivalent is $1.1\overline{36}$, his answer was incorrect.

b. To find the error that Marcos made, write $1.1\overline{36}$ as a fraction.

$$x = 1.1\overline{36}$$
$$10x = 11.\overline{36}$$
$$1000x = 1136.\overline{36}$$
$$1000x - 10x = 1136.\overline{36} - 11.\overline{36}$$
$$990x = 1125$$
$$x = \frac{1125}{990} = \frac{25}{22}$$

Since $1.1\overline{36} = \frac{25}{22}$, Marcos most likely divided the denominator, 25, by the numerator, 22.

Rubric
1 point for the correct decimal equivalent; 1 point for deciding that Marcos gave an incorrect answer; 1 point for writing $1.1\overline{36}$ as a fraction; 1 point for describing what mistake Marcos likely made

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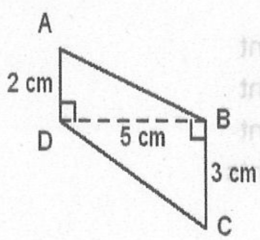
8.NS.2 Answers

1. C
2. C
3. D
4. B, C
5. D, F
6. a. Yes
   b. Yes
   c. No
   d. No
   e. No
7. The lengths of the vertical sides of quadrilateral *ABCD* are the difference in the *y*-coordinates of *A* and *D*, or 2 cm, and the difference in the *y*-coordinates of *B* and *C*, or 3 cm. Also, by connecting points *B* and *D*, you divide the quadrilateral into two right triangles. Since the common leg is horizontal, its length is the difference in the *x*-coordinates of *B* and *D*, or 5 cm. Removing the figure from the coordinate plane, you now have the following information:



Apply the Pythagorean theorem to $\triangle ABD$ to find the length *AB*:

$(AB)^2 = 2^2 + 5^2$

$(AB)^2 = 4 + 25$

$(AB)^2 = 29$

$AB = \sqrt{29} \approx 5.4$ cm

Apply the Pythagorean theorem to $\triangle BCD$ to find the length *CD*:

$(CD)^2 = 3^2 + 5^2$

$(CD)^2 = 9 + 25$

$(CD)^2 = 34$

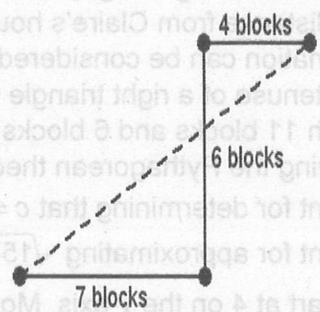
$CD = \sqrt{34} \approx 5.8$ cm

The perimeter of quadrilateral *ABCD*, to the nearest 0.1 cm, is $5.4 + 3 + 5.8 + 2 = 16.2$ cm.

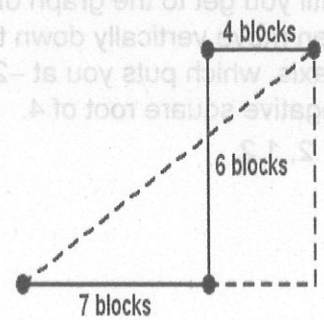
Rubric

1 point for dividing the figure into two right triangles and finding the lengths of the vertical legs; 1 point each for approximating the lengths *AB* and *CD*; 1 point for finding the perimeter

8. Claire's path is shown in the diagram below, with the straight-line distance from her house to her destination shown with a dotted line.



By forming a right triangle using the dashed line as the hypotenuse, you can determine that the lengths of the legs are $7 + 4 = 11$ blocks and 6 blocks.



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Apply the Pythagorean theorem to find the length of the hypotenuse.

$11^2 + 6^2 = c^2$

$121 + 36 = c^2$

$157 = c^2$

$\sqrt{157} = c$

157 falls between the two perfect squares 144 and 169, so

$144 < 157 < 169$

$\sqrt{144} < \sqrt{157} < \sqrt{169}$

$12 < \sqrt{157} < 13$

$12.5^2 = 156.25$

$12.6^2 = 158.76$

$12.5 < \sqrt{157} < 12.6$

Since $12.55^2 = 157.5025 > 157$, round down. The straight-line distance from Claire's house to her destination is about 12.5 blocks.

Rubric

1 point for recognizing that the straight-line distance from Claire's house to her destination can be considered the hypotenuse of a right triangle with legs of length 11 blocks and 6 blocks; 1 point for applying the Pythagorean theorem; 1 point for determining that $c = \sqrt{157}$; 1 point for approximating $\sqrt{157}$ as 12.5

9. a. Start at 4 on the y-axis. Move horizontally to the right until you get to the graph of $y = x^2$; then move vertically down to the x-axis, which puts you at 2, the positive square root of 4. Go back and start at 4 on the y-axis. Move horizontally to the left until you get to the graph of $y = x^2$; then move vertically down to the x-axis, which puts you at –2, the negative square root of 4.

   b. –1.2, 1.2

   c. If you start at 0 on the y-axis, you do not need to move horizontally to get to the graph of $y = x^2$, nor do you need to move vertically to get to the x-axis. So, the only square root of 0 is 0.

Rubric

1 point for each part

10. a. $\sqrt{0.5} \approx \sqrt{0.49} = \sqrt{0.7^2} = 0.7$

   b. No; it shows that $\sqrt{0.5}$ is greater than 0.5, not less than 0.5.

   c. If a positive number is greater than 1, then the number is greater than its square root.

   d.

Let $n > 0$.	Start with a positive number.
$n < 1$	Assume the number is less than 1.
$n^2 < n$	Multiply both sides by n.
$\sqrt{n^2} < \sqrt{n}$	The greater number has the greater square root.
$n < \sqrt{n}$	The square root of a positive number squared is just the number.

Rubric

a. 1 point

b. 1 point

c. 1 point

d. 3 points

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8.EE.1 Answers

1. C
2. B
3. D
4. A
5. B
6. B, F
7. A, D, E, F
8. a. No
   b. Yes
   c. No
   d. No
   e. Yes
9. The missing exponent is 7.
   Since 5^{11} and $5^{?}$ are being divided, the missing exponent is a number that, when subtracted from 11, equals 4. The missing exponent is 7 because $11 - 7 = 4$.

   Rubric
   1 point for answer; 1 point for explanation
10. The missing exponent is –6.
    The equation simplifies to $7^{?} \cdot 7^{20} = 7^{14}$ because $(7^5)^4 = 7^{5 \cdot 4} = 7^{20}$.

    Since $7^{?}$ and 7^{20} are being multiplied, the missing exponent is a number that, when added to 20, equals 14. The missing exponent is –6 because $-6 + 20 = 14$.

    Rubric
    1 point for answer; 1 point for explanation
11. Yes.
    When raising a power to a power, multiply the exponents.

    $(17^3)^4 \cdot 17^{-4} = 17^{3 \cdot 4} \cdot 17^{-4} = 17^{12} \cdot 17^{-4}$

    When multiplying powers with the same base, add the exponents.

    $17^{12} \cdot 17^{-4} = 17^{12 + (-4)} = 17^8$

    Rubric
    1 point for answer; 2 points for explanation
12. No.
    When raising a power to a power, multiply the exponents.

    $\frac{3^{13}}{(3^5)^3} = \frac{3^{13}}{3^{5 \cdot 3}} = \frac{3^{13}}{3^{15}}$

    When dividing powers with the same base, subtract the exponents.

    $\frac{3^{13}}{3^{15}} = 3^{13-15} = 3^{-2}$

    Since $a^{-n} = \frac{1}{a^n}$, $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$.

    Thus, $\frac{3^{13}}{(3^5)^3}$ is not equal to 9.

    Rubric
    1 point for answer; 2 points for explanation
13. Possible answers: $2^2 \cdot 2^4$, $\frac{2^8}{2^2}$, and $(2^2)^3$

    When multiplying powers with the same base, add the exponents.

    $2^2 \cdot 2^4 = 2^{2+4} = 2^6$

    When dividing powers with the same base, subtract the exponents.

    $\frac{2^8}{2^2} = 2^{8-2} = 2^6$

    When raising a power to a power, multiply the exponents and keep the same base.

    $(2^2)^3 = 2^{2 \cdot 3} = 2^6$

    Rubric
    1 point for each answer; 1 point for each explanation

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14. a. $\frac{3^2 \cdot 3^8}{(3^3)^2} = \frac{3^{2+8}}{3^{3 \cdot 2}} = \frac{3^{10}}{3^6} = 3^{10-6} = 3^4$

The expression is equivalent to 3^4.

b. $\left(\frac{3^9}{3^5 \cdot 3^6}\right)^2 = \left(\frac{3^9}{3^{5+6}}\right)^2 = \left(\frac{3^9}{3^{11}}\right)^2 =$

$(3^{9-11})^2 = (3^{-2})^2 = 3^{-2 \cdot 2} = 3^{-4}$

The expression is not equivalent to 3^4.

Rubric

a. 1 point for correct application of each of the three properties used to simplify the expression

b. 1 point for correct application of each of the three properties used to simplify the expression

15. Student C got the correct answer.

Student A incorrectly wrote that $\frac{3^3}{9}$ is $\frac{1^3}{3}$. The student should have either carried out the exponentiation to get $\frac{3^3}{9} = \frac{27}{9} = 3$ or written 9 as 3^2 to use the quotient of powers property: $\frac{3^3}{9} = \frac{3^3}{3^2} = 3^1 = 3$.

Student B incorrectly used the product of powers property when replacing $3^3 \cdot 3^{-1}$ with 9^2. The student should have kept the base 3 and added the exponents to get $3^3 \cdot 3^{-1} = 3^2$.

Rubric

1 point for stating that Student C is correct;
1 point for stating that Student A is incorrect;
1 point for correctly pointing out Student A's error;
1 point for stating that Student B is incorrect;
1 point for correctly pointing out Student B's error

16. a. Divide the radius of the Sun by the radius of Earth. When dividing exponents with the same base, subtract the exponents.

$\frac{10^9}{10^7} = 10^{9-7} = 10^2$

The radius of the Sun is roughly $10^2 = 100$ times that of Earth, so the diameter of the Sun is roughly 100 times that of Earth. Thus, about 100 copies of Earth have to line up end-to-end in order to stretch across the face of the Sun.

b. When finding the volume of a sphere, the radius of the sphere is cubed. So, cube the ratio of the radius of the Sun to the radius of Earth. When raising a power to a power, multiply the exponents.

$(10^2)^3 = 10^6$

So, roughly $10^6 = 1{,}000{,}000$ copies of Earth could fit inside the Sun.

Rubric

a. 1 point for answer; 1 point for explanation

b. 1 point for answer; 1 point for explanation

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8.EE.2 Answers

1. B
2. A
3. B
4. A
5. B
6. B, C, E
7. A, C, F
8. G
9. H
10. E
11. D
12. A
13. F
14. First, find the area A_L of the living room.

$A_L = \ell w$

$= 15 \times 12$

$= 180$

Next, find the area A_R of the rug by multiplying the area A_L of the living room by $\frac{5}{9}$.

$A_R = \frac{5}{9}(180)$

$= 100$

The area of the rug is 100 ft^2. To find the side length s of the rug, take the square root of 100.

$s^2 = 100$

$s = \sqrt{100}$

$s = 10$

The side length of the rug is 10 ft.

Rubric
1 point for the area of the living room;
1 point for the area of the rug;
1 point for the side length of the rug

15. First, find n by taking the cube root of both sides of the equation.

$n^3 = 64$

$n = \sqrt[3]{64}$

$n = 4$

Next, find n^2 by squaring the value of n.

$n = 4$

$n^2 = 4^2$

$n^2 = 16$

Rubric
1 point for n; 1 point for explanation;
1 point for n^2

16. The side length of the first piece of wrapping paper is $\sqrt{36} = 6$ in.

The side length of the second piece of wrapping paper is 6 + 1 = 7 in.

The area of the second piece of wrapping paper is $7^2 = 49$ in^2.

Rubric
1 point for each side length;
1 point for the area of the second piece

17. Find the volume V_P of the prism.

$V_P = \ell wh$

$= 2 \times 4 \times 8$

$= 64$

The volume of the rectangular prism is 64 in^3. Now, find the side length s of the cube with the same volume.

$V_C = s^3$

$64 = s^3$

$\sqrt[3]{64} = s$

$4 = s$

The side length of a cube with volume 64 in^3 is 4 in.

Rubric
1 point for finding volume of prism;
1 point for finding side length of cube

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18. First, simplify the right side of the equation.

$x^2 = \sqrt[3]{125}$

$x^2 = \sqrt[3]{5^3}$

$x^2 = 5$

Next, apply the definition of a square root.

$x^2 = 5$

$x = \pm\sqrt{5}$

Rubric

1 point for work shown; 1 point for each answer

19. Begin by finding the square roots and cube roots of 1, 64, and 729.

$\sqrt{1} = \sqrt{1^2} = 1 = 1^3$

$\sqrt[3]{1} = \sqrt[3]{1^3} = 1 = 1^2$

$\sqrt{64} = \sqrt{8^2} = 8 = 2^3$

$\sqrt[3]{64} = \sqrt[3]{4^3} = 4 = 2^2$

$\sqrt{729} = \sqrt{27^2} = 27 = 3^3$

$\sqrt[3]{729} = \sqrt[3]{9^3} = 9 = 3^2$

To find the next number that is both a perfect square and a perfect cube, let $n = 4$. Find n^3, and then square that result.

$n = 4$

$(n^3)^2 = (4^3)^2$

$= 64^2$

$= 4096$

The next largest number that is a perfect square and a perfect cube is 4096 because $64^2 = 4096$ and $16^3 = 4096$.

Rubric

1 point each for finding the square roots and cube roots of 1, 64, and 729;
1 point for finding the next number that is both a perfect square and perfect cube;
1 point for showing the appropriate work

20. a. Take the square root of each area to find the side length of each square.

$A_R = s^2 \qquad A_B = s^2$

$25 = s^2 \qquad 16 = s^2$

$\sqrt{25} = s \qquad \sqrt{16} = s$

$5 = s \qquad 4 = s$

The side length of the red square is 5 inches. The side length of the blue square is 4 inches.

b. $12(5) + 12(4) = 60 + 48 = 108$ inches, or 9 feet

Rubric

a. 1 point for each side length

b. 1 point for answer (in inches or feet)

21. a. First, find the side length s by taking the cube root of 27.

$s = \sqrt[3]{27}$

$= 3$

Next, find the area of a glass face of the cube by squaring the side length.

$A_{face} = s^2$

$= 3^2$

$= 9$

Multiply the area of a face by the number of faces.

$A_{glass} = 9 \times 5$

$= 45$

So, 45 ft^2 of glass is required to make the fish tank.

b. Since there is no top, the number of edges that need to be reinforced is 8. To find the total length of metal framing needed, multiply the side length by the number of edges.

8×3 ft $= 24$ ft

Rubric

a. 1 point for the side length; 1 point for the area of a face; 1 point for the area of glass needed

b. 1 point for the total length of metal framing needed

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8.EE.3 Answers

1. C
2. B
3. A
4. A
5. D
6. A, D, E
7. C, D
8. C
9. D
10. A
11. B
12. F
13. 32.6 million = 32,600,000

$\frac{32,600,000}{3.26} = 10,000,000 = 10^7$

$32,600,000 = 3.26 \times 10^7$

Rubric

1 point for work shown; 1 point for the correct answer

14.

$\frac{5\times10^{15}}{2\times10^{12}} = \frac{5}{2}\times\frac{10^{15}}{10^{12}}$

$= 2.5\times10^3$

$= 2500$

Alea's estimate is 2500 times greater than Carlos's estimate.

Rubric

1 point for work shown; 1 point for correct answer

15. Write 0.00000057652 in scientific notation: $0.00000057652 = 5.7652 \times 10^{-7}$

Notice that 5.764×10^{-7} and 5.7652×10^{-7} have the same power of 10. Since $5.7652 > 5.764$, 5.7652×10^{-7} is the greater number.

Rubric

1 point for correct reasoning; 1 point for the correct answer

16. Begin by finding the area of the colony of bacteria in square meters. Then compare that result with the given area of the colony.

Area of colony = area of bacterium $\times 100$

$= (7.3\times10^{-12})\times100$

$= (7.3\times10^{-12})\times10^2$

$= 7.3\times10^{-10}$

Divide the area of the colony given in the statement of the problem by the area of the colony in square meters.

$\frac{7.3\times10^{-6}}{7.3\times10^{-10}} = \left(\frac{7.3}{7.3}\times\frac{10^{-6}}{10^{-10}}\right)$

$= 1\times10^{-6-(-10)}$

$= 1\times10^4$

$= 10,000$

Since the number given for the area in the statement of the problem is 10,000 times greater than the area calculated in square meters, the units cannot be square meters. Find how many square centimeters and square millimeters are in 1 square meter.

1 m = 100 cm, so

$1\text{ m}^2 = 100\text{ cm} \times 100\text{ cm} = 10,000\text{ cm}^2$;

1 m = 1000 mm, so

$1\text{ m}^2 = 1000\text{ mm} \times 1000\text{ mm} = 1,000,000\text{ mm}^2$

Since there are 10,000 cm^2 in 1 m^2, the units are in square centimeters.

Rubric

2 points for the explanation; 1 point for the correct units

17. a. $\frac{5.3\times10^5}{1.06\times10^4} = \frac{5.3}{1.06}\times\frac{10^5}{10^4} = 5\times10^1 = 50$, so the population of the city is 50 times the population of the town.

b. $\frac{6.28\times10^6}{1.256\times10^5} = \frac{6.28}{1.256}\times\frac{10^6}{10^5} = 5\times10^1 =$ 50, so the bacteria level in the city's water sample is 50 times the bacteria level in the town's water sample.

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c. Since the city has a population that is 50 times greater than the town and the city has 50 times more bacteria in its water sample, the bacteria in the water is proportional to the population.

Rubric

a. 1 point

b. 1 point

c. 2 points

18. The masses must be measured in the same units to be compared, so convert the mass of object A to milligrams.

$1 \text{ kg} = 10^3 \text{ g} = 10^6 \text{ mg}$

$$\left(1.325\times10^{-4}\text{ kg}\right)\left(10^6\ \frac{\text{mg}}{\text{kg}}\right) = 1.325\times10^2\text{ mg}$$

$$\frac{\text{mass of object B}}{\text{mass of object A}} = \frac{3.3125\times10^3}{1.325\times10^2} = \frac{3.3125}{1.325}\times\frac{10^3}{10^2} = 2.5\times10^1 = 25$$

The mass of object B is 25 times greater than the mass of object A, so Ana is correct.

To find Michael's error, compare 25,000,000 and 25.

$$\frac{25{,}000{,}000}{25} = 1{,}000{,}000$$

Because 25,000,000 is 1,000,000 times greater than 25 and there are 1,000,000 mg in 1 kg, Michael most likely did not convert the masses of the objects to the same unit.

Rubric

1 point for comparing the masses;
1 point for deciding who is correct;
1 point for identifying the error

19. a. $57{,}910{,}000 = 5.791\times10^7$
$149{,}600{,}000 = 1.496\times10^8$
$1{,}429{,}400{,}000 = 1.4294\times10^9$

b.

$$\frac{1.496\times10^8}{5.791\times10^7} = \frac{1.496}{5.791}\times\frac{10^8}{10^7} \approx 0.2583\times10^1 = 2.583$$

c.

$$\frac{1.4294\times10^9}{5.791\times10^7} = \frac{1.4294}{5.791}\times\frac{10^9}{10^7} \approx 0.2468\times10^2 = 24.68$$

d. The comparisons in parts b and c use Mercury's distance from the Sun as a "yardstick" in the sense that Earth is about 2.6 "Mercury distances" from the Sun and Saturn is about 25 "Mercury distances" from the Sun. So, Saturn is about 10 times as far from the Sun as Earth is. This is confirmed by a direct comparison of the actual distances.

$$\frac{1.4294\times10^9}{1.496\times10^8} = \frac{1.4294}{1.496}\times\frac{10^9}{10^8} \approx 1\times10^1 = 10$$

Rubric

a. 1 point for each number written in scientific notation

b. 1 point

c. 1 point

d. 1 point

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8.EE.4 Answers

1. B
2. D
3. B
4. A
5. A
6. B, D, F
7. A, C, E
8. C
9. F
10. A
11. D
12. B
13. $(1.2\times10^{-5}\text{ m})(1.5\times10^{-4}\text{ m})$

$$=(1.2\times1.5)\times10^{-5-4}\text{ m}^2$$
$$=1.8\times10^{-9}\text{ m}^2$$
$$=(1.8\times10^{-9}\text{ m}^2)\left(\frac{1\text{ mm}}{10^{-3}\text{ m}}\right)^2$$
$$=(1.8\times10^{-9}\text{ m}^2)\left(\frac{1\text{ mm}^2}{10^{-6}\text{ m}^2}\right)$$
$$=1.8\times10^{-9-(-6)}\text{ mm}^2$$
$$=1.8\times10^{-3}\text{ mm}^2$$

Rubric

1 point for multiplying correctly; 1 point for correct answer with correct units

14. Simplify the numerator and the denominator, and then divide.

$$\frac{3\times10^{-3}+6\times10^{-2}}{(7\times10^4)(3\times10^8)}=\frac{3\times10^{-3}+60\times10^{-3}}{(7\times10^4)(3\times10^8)}$$
$$=\frac{(3+60)\times10^{-3}}{(7\times10^4)(3\times10^8)}$$
$$=\frac{63\times10^{-3}}{(7\times3)\times10^{4+8}}$$
$$=\frac{63\times10^{-3}}{21\times10^{12}}$$
$$=\frac{63}{21}\times10^{-3-12}$$
$$=3\times10^{-15}$$

Rubric

1 point for simplifying the numerator correctly; 1 point for simplifying the denominator correctly; 1 point for simplifying the quotient correctly

15. a. $(2\times10^7)(1.55\text{ mm})=3.1\times10^7\text{ mm}$

b. $(3.1\times10^7\text{ mm})\left(\frac{1\text{ m}}{10^3\text{ mm}}\right)=3.1\times10^4\text{ m}$

$(3.1\times10^4\text{ m})\left(\frac{1\text{ km}}{10^3\text{ m}}\right)=3.1\times10^1\text{ km}$

$=31\text{ km}$

c. Kilometers is the most appropriate unit for the height of the stack because 3.1×10^7 mm and 3.1×10^4 m are very large numbers that cannot be visualized easily.

Rubric

1 point for each part

16. $V\approx3.35\times10^{-20}\text{ m}^3$

Rubric

1 point for the correct coefficient; 1 point for the correct power of 10

17. a. In 10 years, the person consumes

$\left(8\frac{\text{glasses}}{\text{day}}\right)\left(3\times10^2\frac{\text{mL}}{\text{glass}}\right)\bullet\left(365\frac{\text{days}}{\text{years}}\right)(10\text{ years})=$

8.76×10^6 mL of water.

b. $8.76\times10^6\text{ mL}=8.76\times10^3\text{ L}=8760\text{ L}$, so the more appropriate unit is liters. When represented in milliliters, the number is very large and difficult to visualize.

Rubric

a. 1 point for the correct coefficient; 1 point for the correct power of 10

b. 1 point for the correct unit; 1 point for the explanation

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18. To decide which flash drive to purchase, first find total number of bytes you need.

Total number of bytes

$= 9.65 \times 10^7 + 12(7.5 \times 10^7)$

$= 9.65 \times 10^7 + 90 \times 10^7$

$= 99.65 \times 10^7$

$= 9.965 \times 10^8$

Next, write 750 megabytes and 2 gigabytes in scientific notation.

$750 \text{ megabytes} = 750 \times 10^6 \text{ bytes}$

$= 7.5 \times 10^8 \text{ bytes}$

$2 \text{ gigabytes} = 2 \times 10^9 \text{ bytes}$

Since $9.965 \times 10^8 > 7.5 \times 10^8$, you need to purchase the 2 gigabyte flash drive.

Rubric

2 points for the total number of bytes needed; 1 point for comparing the values correctly; 1 point for identifying the correct flash drive

19. a. $\dfrac{7.2 \times 10^2 + 9.63 \times 10^3}{9 \times 10^9}$

= 1.15E–6 (obtained from a calculator)

$= 1.15 \times 10^{-6}$

$= 0.00000115$

b. Alessandro correctly used his calculator to simplify the expression, but incorrectly interpreted the exponent –6 and wrote 6 zeros rather than writing 5 zeros to move the decimal point 6 places to the left, so Alessandro's answer in standard form had one extra zero.

Rubric

2 points for each part

20. Calculate the average densities of the Sun and Earth. Then compare them. To find the average density of the Sun, first find the volume.

$$V = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}(3.14)\left(\frac{1{,}390{,}000}{2} \text{ km}\right)^3$$

$$\approx 1.405 \times 10^{18} \text{ km}^3$$

Next, find the average density.

$$D = \frac{m}{V}$$

$$= \frac{1.989 \times 10^{30} \text{ kg}}{1.405 \times 10^{18} \text{ km}^3}$$

$$\approx 1.416 \times 10^{12} \text{ kg/km}^3$$

Use the same process to find the average density of Earth. First, find the volume.

$$V = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}(3.14)\left(\frac{12{,}756}{2} \text{ km}\right)^3$$

$$\approx 1.086 \times 10^{12} \text{ km}^3$$

Next, find the average density.

$$D = \frac{m}{V}$$

$$= \frac{5.972 \times 10^{24} \text{ kg}}{1.086 \times 10^{12} \text{ km}^3}$$

$$\approx 5.499 \times 10^{12} \text{ kg/km}^3$$

Since $5.499 \times 10^{12} > 1.416 \times 10^{12}$, the average density of Earth is greater.

Rubric

1 point for the volume of the Sun;

1 point for the density of the Sun;

1 point for the volume of Earth;

1 point for the density of Earth;

1 point for the correct comparison

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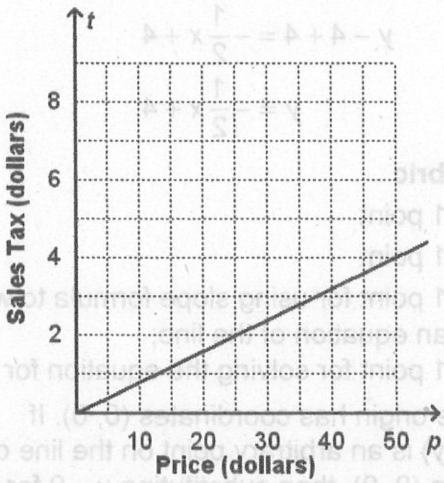
8.EE.5 Answers

1. C
2. B
3. A
4. A, C, D
5.



The slope of the line is 0.08. The slope represents a sales tax of $0.08 for every dollar spent, or an 8% sales tax rate.

Rubric
1 point for correct graph; 1 point for slope; 1 point for interpretation of slope

6. To determine the unit rate for the remote-controlled truck, divide the given distance by the given time.

$$\frac{\text{Distance}}{\text{Time}} = \frac{108}{6} = 18$$

So, the remote-controlled truck has a unit rate of 18 feet per second.

To determine the unit rate for the remote-controlled car, divide the distance by the time for any data pair in the table.

$$\frac{\text{Distance}}{\text{Time}} = \frac{63}{3} = 21$$

So, the remote-controlled car has a unit rate of 21 feet per second.

Since the remote-controlled car has the greater unit rate, the remote-controlled car travels faster.

Rubric
1 point for answer; 2 points for explanation

7. Brand R consumes 160 watts per hour.

Divide 435 by 3 to find the number of watts per hour that brand S consumes.

$$\frac{\text{Watts}}{\text{Time}} = \frac{435}{3} = 145$$

Brand S consumes 145 watts per hour, so it consumes less power than brand R.

Rubric
1 point for answer; 2 points for explanation

8. Find Pedro's reading rate by dividing the given number of pages by the given amount of time.

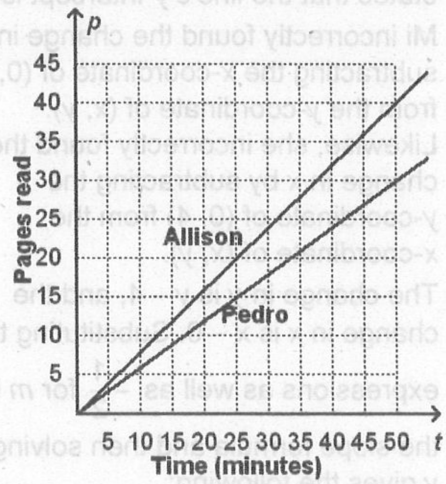
$$\frac{\text{Pages}}{\text{Time}} = \frac{24}{40} = 0.6$$

So, Pedro reads 0.6 page per minute. The slope of the line for Pedro is 0.6.

Find Allison's reading rate by dividing the number of pages by the amount of time for any data pair in the table.

$$\frac{\text{Pages}}{\text{Time}} = \frac{12}{15} = 0.8$$

So, Allison reads 0.8 page per minute. The slope of the line for Allison is 0.8.



Allison reads faster than Pedro because her reading rate is greater.

Rubric
0.5 point for each line; 0.5 point for each label; 1 point for stating that Allison reads faster; 1 point for explanation

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8.EE.6 Answers

1. B
2. C
3. A
4. D
5. The origin has coordinates (0, 0), so the slope of the line is $m = \frac{8-0}{4-0} = \frac{8}{4} = 2$. If (x, y) is an arbitrary point on the line other than (0, 0), then substituting $y - 0$ for the change in y, $x - 0$ for the change in x, and 2 for m in the slope formula $\frac{\text{change in } y}{\text{change in } x} = m$ gives $\frac{y-0}{x-0} = 2$ as an equation of the line. Solving the equation for y gives $y = 2x$.

   Rubric
   1 point for finding the slope of the line;
   1 point for deriving an equation of the line from the slope formula; 1 point for rewriting the equation
6. a. Mi's equation is in the form $y = mx + b$, which tells you that the line whose equation she has found has y-intercept $b = 2$. But the problem states that the line's y-intercept is 4.

   b. Mi incorrectly found the change in y by subtracting the x-coordinate of (0, 4) from the y-coordinate of (x, y). Likewise, she incorrectly found the change in x by subtracting the y-coordinate of (0, 4) from the x-coordinate of (x, y).

   c. The change in y is $y - 4$, and the change in x is $x - 0$. Substituting these expressions as well as $-\frac{1}{2}$ for m into the slope formula and then solving for y gives the following:

$$\frac{\text{change in } y}{\text{change in } x} = m$$

$$\frac{y-4}{x-0} = -\frac{1}{2}$$

$$\frac{y-4}{x} \bullet x = -\frac{1}{2} \bullet x$$

$$y - 4 + 4 = -\frac{1}{2}x + 4$$

$$y = -\frac{1}{2}x + 4$$

   Rubric
   a. 1 point
   b. 1 point
   c. 1 point for using slope formula to write an equation of the line;
   1 point for solving the equation for y
7. The origin has coordinates (0, 0). If (x, y) is an arbitrary point on the line other than (0, 0), then substituting $y - 0$ for the change in y and $x - 0$ for the change in x in the slope formula $\frac{\text{change in } y}{\text{change in } x} = m$ gives $\frac{y-0}{x-0} = m$ as an equation of the line. Solving the equation for y gives $y = mx$.

   Rubric
   1 point for deriving the equation; 1 point for rewriting the equation
8. If (x, y) is an arbitrary point on the line other than (0, b), then substituting $y - b$ for the change in y and $x - 0$ for the change in x in the slope formula $\frac{\text{change in } y}{\text{change in } x} = m$ gives $\frac{y-b}{x-0} = m$ as an equation of the line. Solving the equation for y gives $y = mx + b$.

   Rubric
   1 point for deriving the equation;
   1 point for rewriting the equation

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9. a. Possible answer:

To map the smaller triangle onto the larger triangle, translate the smaller triangle 2 units to the right and 3 units down. This translation maps the smaller triangle's vertex at (–4, 6) to the larger triangle's vertex at (–2, 3). Dilate the image of the smaller triangle using a scale factor of 2 and the point (–2, 3) as the center of dilation. The image of the smaller triangle after the translation and dilation completely coincides with the larger triangle, so the triangles must be similar.

b. For the smaller triangle, the vertical leg has a directed length of –3 because the movement from the vertex at (–2, 3) to the vertex at (–4, 3) is downward, while the horizontal leg has a directed length of 2 because the movement from the vertex at (–4, 3) to the vertex at (–2, 3) is to the right. So, the slope of the line based on this triangle is $-\frac{3}{2}$.

For the larger triangle, the vertical leg has a directed length of –6 while the horizontal leg has a directed length of 4. So, the slope of the line based on this triangle is $-\frac{6}{4}$, or $-\frac{3}{2}$.

The two slopes should be the same because the triangles were shown to be similar. That means that the directed lengths of the legs of the larger triangle are some scale factor (in this case, 2) times the directed lengths of the corresponding legs of the smaller triangle. When you form the ratio of the directed length of the vertical leg to the directed length of the horizontal leg for the larger triangle, the scale factor divides out, producing the same ratio as that obtained for the smaller triangle.

c. You can show that through a sequence of transformations the original smaller right triangle is similar to any right triangle with a vertical leg, a horizontal leg, and a portion of the line as the hypotenuse. This means that you can multiply the directed lengths of the legs of the smaller right triangle by some nonzero scale factor *k* to obtain the directed lengths of the corresponding legs of the new right triangle. In other words, for the new right triangle, the directed length of the vertical leg is $-3k$, while the directed length of the horizontal leg is $2k$. So, the slope of the line based on the new right triangle is $\frac{-3k}{2k}$, or $-\frac{3}{2}$. This argument demonstrates that the slope of the line is always $-\frac{3}{2}$.

Rubric

a. 1 point for a correct translation;
1 point for a correct dilation

b. 1 point for correct slope using smaller triangle;
1 point for correct slope using larger triangle;
1 point for explaining why the slopes are the same

c. 1 point for identifying the directed lengths of the legs of the arbitrary right triangle;
1 point for showing that the slope remains the same

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8.EE.7a Answers

1. B
2. A
3. C
4. C
5. D
6. C, E
7. A, C, F
8. a. One solution
   b. Infinitely many solutions
   c. One solution
   d. One solution
   e. No solutions
9. Possible answer: $6x - 28 = 7(x - 4) - x$

$$6x - 28 = 7(x - 4) - x$$
$$6x - 28 = 7x - 28 - x$$
$$6x - 28 = 6x - 28$$
$$-28 = -28$$

The statement $-28 = -28$ is true for any value of x, so any real number is a solution of the equation. Since there are infinitely many real numbers, the equation has infinitely many solutions.

Rubric
1 point for an equation that satisfies the criteria; 1 point for solving equation; 1 point for number of solutions; 1 point for explanation

10. Possible answer: $-2x + 7 = -2x$

$$-2x + 7 = -2x$$
$$-2x + 2x + 7 = -2x + 2x$$
$$7 = 0$$

The statement $7 = 0$ is false no matter what the value of x is, so the equation has no solutions.

Rubric
1 point for an equation that satisfies the criteria; 1 point for showing work; 1 point for explanation

11. Possible answer: $4x + 14 = -x + 9$

$$4x + 14 = -x + 9$$
$$5x + 14 = 9$$
$$5x = -5$$
$$x = -1$$

Rubric
2 points for an equation that satisfies the criteria; 1 point for simplifying equation to the form $x = a$

12. $-x + 5(x + 6) = 4x + 30$

$$-x + 5x + 30 = 4x + 30$$
$$4x + 30 = 4x + 30$$
$$30 = 30$$

Sarah did not distribute correctly; each term of $x + 6$ must be multiplied by 5. There are infinitely many solutions because the equation is equivalent to a statement that is true for any value of x.

Rubric
2 points for correct solving; 1 point for identifying error; 1 point for correct number of solutions

13. Mark and Zoe have hiked the same distance when the equation $2t + 100 = 2t$ is true.

Solve the equation.

$$2t + 100 = 2t$$
$$100 = 0$$

Since the equation is equivalent to $100 = 0$, which is not true for any value of t, the equation has no solutions. Therefore, there is no time during the hike when Mark and Zoe have hiked the same distance.

Rubric
1 point for setting the expressions for the distances equal to each other; 1 point for solving; 1 point for determining there are no solutions; 1 point for correct conclusion

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14. a. Solve the equation.

$$8x - 7 = 2(2x + 7) - 5$$
$$8x - 7 = 4x + 14 - 5$$
$$8x - 7 = 4x + 9$$
$$8x - 4x - 7 = 4x - 4x + 9$$
$$4x - 7 + 7 = 9 + 7$$
$$\frac{4x}{4} = \frac{16}{4}$$
$$x = 4$$

The result $x = 4$ is true only when the value of x is 4. So, the equation has one solution.

b. Possible answer: Change the 2 outside the parentheses to 4, and solve the equation.

$$8x - 7 = 4(2x + 7) - 5$$
$$8x - 7 = 8x + 28 - 5$$
$$8x - 7 = 8x + 23$$
$$8x - 8x - 7 = 8x - 8x + 23$$
$$-7 = 23$$

The result $-7 = 23$ is not true for any value of x. So, the equation has no solutions.

c. Possible answer: Change the 5 to 35, and solve.

$$8x - 7 = 4(2x + 7) - 35$$
$$8x - 7 = 8x + 28 - 35$$
$$8x - 7 = 8x - 7$$
$$8x - 8x - 7 = 8x - 8x - 7$$
$$-7 = -7$$

The result $-7 = -7$ is true for any value of x. So, the equation has infinitely many solutions.

Rubric

a. 1 point for the number of solutions; 1 point for work and explanation

b. 1 point for a change that results in a different number of solutions; 1 point for the number of solutions; 1 point for work and explanation

c. 1 point for a change that results in a different number of solutions; 1 point for the number of solutions; 1 point for work and explanation

15. The equation has no solutions when $a = c$ and $b \neq cd$. Substitute c for a and solve for x as shown.

$$cx + b = c(x + d)$$
$$cx + b = cx + cd$$
$$cx - cx + b = cx - cx + cd$$
$$b = cd$$

If $b \neq cd$, the equation has no solutions because no value of x makes the equation a true statement.

The equation has one solution when $a \neq c$. Solve for x as shown.

$$ax + b = c(x + d)$$
$$ax + b = cx + cd$$
$$ax - cx + b = cx - cx + cd$$
$$ax - cx + b - b = cd - b$$
$$\frac{(a - c)x}{a - c} = \frac{cd - b}{a - c}$$
$$x = \frac{cd - b}{a - c}$$

The expression $\frac{cd - b}{a - c}$ is defined when $a \neq c$. So, the equation has one solution because only the number $\frac{cd - b}{a - c}$ results in a true statement when substituted for x.

The equation has infinitely many solutions when $a = c$ and $b = cd$. Substitute c for a and solve for x as shown.

$$cx + b = c(x + d)$$
$$cx + b = cx + cd$$
$$cx - cx + b = cx - cx + cd$$
$$b = cd$$

If $b = cd$, the equation has infinitely many solutions because every value of x makes the equation a true statement.

Rubric

1 point for each set of circumstances; 1 point for each explanation

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8.EE.7b Answers

1. C
2. B
3. B
4. C, E
5.

$$-\frac{3}{5}(x-10)=\frac{6}{5}x+2$$
$$-\frac{3}{5}x+6=\frac{6}{5}x+2$$
$$-\frac{3}{5}x-\frac{6}{5}x+6=\frac{6}{5}x-\frac{6}{5}x+2$$
$$-\frac{9}{5}x+6-6=2-6$$
$$-\frac{9}{5}x=-4$$
$$x=\frac{20}{9}$$

Rubric

1 point for showing work; 1 point for answer

6. No, because the solution of $\frac{2}{3}(x-6)+3=4x-4$ is $\frac{9}{10}$, and the solution of $\frac{2}{3}x-6+3=4x-4$ is $\frac{3}{10}$. The only difference between the equations is that the second equation does not have parentheses. Using the distributive property on $\frac{2}{3}(x-6)$ yields $\frac{2}{3}x-4$, not $\frac{2}{3}x-6$ like the second equation.

Rubric

1 point for answering the question; 0.5 point for each solution; 1 point for explanation

7. Substitute 98 for A, 7 for h, and 11 for b_2.

$$98=\frac{1}{2}(7)(b_1+11)$$
$$98=\frac{7}{2}(b_1+11)$$
$$98=\frac{7}{2}b_1+\frac{7}{2}\cdot 11$$
$$98-\frac{77}{2}=\frac{7}{2}b_1+\frac{77}{2}-\frac{77}{2}$$
$$\frac{2}{7}\cdot\frac{119}{2}=\frac{2}{7}\cdot\frac{7}{2}b_1$$
$$17=b_1$$

The length of the base is 17 meters.

Rubric

1 point for showing work; 1 point for answer

8. It would take 100 seconds for Ryan to catch up to Nate, and Ryan would have to swim 125 meters. The equation that models this situation is $\frac{5}{4}t=\frac{4}{5}t+45$, where t is the time, in seconds, after Ryan and Nate start swimming. Solving the equation gives $t=100$. Multiply $\frac{5}{4}$ by 100 to get the distance Ryan swims during those 100 seconds, which is 125 meters.

Rubric

1 point for amount of time; 1 point for distance; 1 point for explanation

9. a.

$$\frac{1}{4}(x-7)+5=\frac{7}{8}x$$
$$\frac{1}{4}x-\frac{7}{4}+5=\frac{7}{8}x$$
$$\frac{1}{4}x+\frac{13}{4}=\frac{7}{8}x$$
$$\frac{1}{4}x-\frac{1}{4}x+\frac{13}{4}=\frac{7}{8}x-\frac{1}{4}x$$
$$\frac{8}{5}\cdot\frac{13}{4}=\frac{8}{5}\cdot\frac{5}{8}x$$
$$\frac{26}{5}=x$$

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b. The least common denominator of the fractions is 8. Begin by multiplying both sides of the equation by 8.

$$8\left(\frac{1}{4}(x-7)+5\right)=8\left(\frac{7}{8}x\right)$$

$$8\cdot\frac{1}{4}(x-7)+8\cdot5=8\cdot\frac{7}{8}x$$

$$2(x-7)+40=7x$$

$$2x-14+40=7x$$

$$2x+26=7x$$

$$2x-2x+26=7x-2x$$

$$\frac{26}{5}=\frac{5x}{5}$$

$$\frac{26}{5}=x$$

The solution is the same one obtained in part a.

c. The multiplication property of equality justifies multiplying both sides of the equation by 8 as a first step. This step is helpful because it removes the fractions from the equation.

Rubric

a. 1 point for using distributive property; 1 point for solution

b. 1 point for multiplying equation by LCD; 1 point for solution

c. 1 point for property; 1 point for explanation

10. Christian's mistake is that he added $\frac{1}{2}x$ to both sides of the equation instead of subtracting $\frac{1}{2}x$ from both sides.

$$\frac{3}{4}x+5=\frac{1}{2}(x-8)$$

$$\frac{3}{4}x+5=\frac{1}{2}x-4$$

$$\frac{3}{4}x-\frac{1}{2}x+5=\frac{1}{2}x-\frac{1}{2}x-4$$

$$\frac{1}{4}x+5-5=-4-5$$

$$\frac{1}{4}x=-9$$

$$x=-36$$

The actual solution is −36.

Rubric

1 point for identifying error; 1 point for correcting error; 1 point for solution; 1 point for showing work

11. a. $1.75h + 12 = 0.50h + 57$

b. The booster club must sell 36 hot dogs to break even.

$$1.75h+12=0.50h+57$$

$$1.75h-0.50h+12=0.50h-0.50h+57$$

$$1.25h+12-12=57-12$$

$$\frac{1.25h}{1.25}=\frac{45}{1.25}$$

$$h=36$$

c. The booster club should charge \$3 for each hot dog if it wants to sell 18 hot dogs to break even.

Substitute p for 1.75 and substitute half of 36, or 18, for h in the equation from part a.

$$(p)(18)+12=0.50(18)+57$$

$$18p+12=9+57$$

$$18p+12-12=66-12$$

$$18p=54$$

$$p=3$$

Rubric

a. 1 point

b. 1 point for showing work; 1 point for answer

c. 1 point for showing work; 1 point for answer

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8.EE.8a Answers

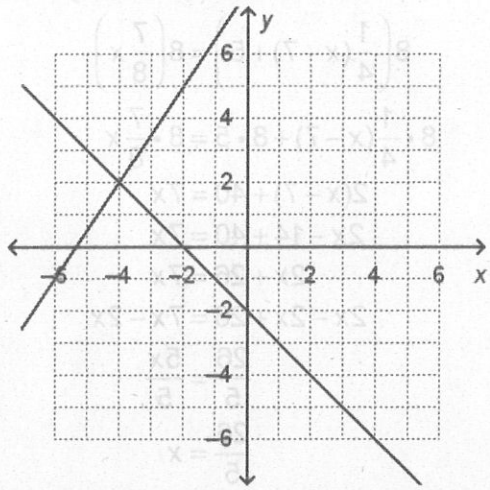
1. C
2. C
3. A
4. B, D
5. Every point that the lines have in common is a solution. There are infinitely many solutions because the lines have infinitely many points in common.

   Rubric
   1 point for the number of solutions;
   1 point for explanation
6. No, because only one line passes through the point (–5, –4).

   Rubric
   1 point for answer; 1 point for explanation
7. Matt is not correct even though the lines do not intersect within the boundaries of the grid shown. The lines are not parallel and will intersect at some point in Quadrant III.

   Rubric
   1 point for answer; 1 point for explanation
8. Yes, because the graph of the system of linear equations $\begin{cases} -2x + y = 1 \\ 4x - 2y = -2 \end{cases}$ consists of two lines that coincide and pass through the points (3, 7) and (6, 13).

   Rubric
   1 point for answer; 1 point for example
9. a. Possible answer:



   b. The pair of lines represents a system of equations with the solution (–4, 2) because the lines intersect at the point (–4, 2).

   Rubric
   a. 2 points
   b. 1 point
10. a. The error is that Chandler thought that the lines intersect at the point (–2, –1). However, the lines intersect at the point (–1, –2).

    b. The graph of $y = -x - 3$ already passes through the point (–2, –1). By translating the graph of $y = 3x + 1$ to the left $1\frac{1}{3}$ units, you will have a line that passes through the point (–2, –1) as well.

    Rubric
    a. 1 point for identifying the error; 1 point for correcting the error
    b. 1 point

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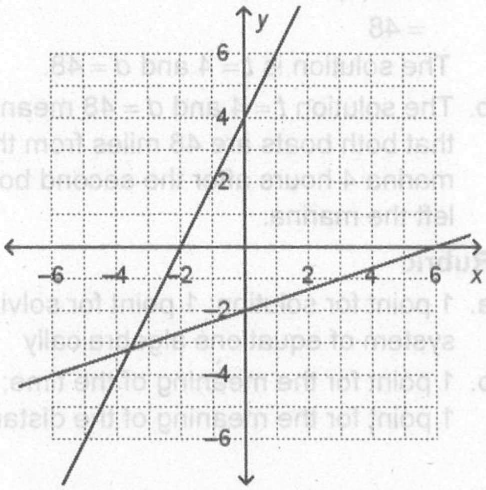
8.EE.8b Answers

1. A
2. B
3. A
4. a. Infinitely many solutions
   b. No solutions
   c. One solution
   d. No solutions
   e. Infinitely many solutions
5.



To the nearest 0.5, the lines appear to intersect at (−3.5, −3). So, an estimate for the solution of the system of linear equations is $x = -3.5$ and $y = -3$.

Rubric

0.5 point for each line; 1 point for estimate of solution of system

6. Solve the equation $-x + y = 4$ for y.

$$-x + y = 4$$
$$y = x + 4$$

Substitute $x + 4$ for y in $5x + 2y = 1$, and solve for x.

$$5x + 2(x + 4) = 1$$
$$5x + 2x + 8 = 1$$
$$7x + 8 = 1$$
$$7x + 8 - 8 = 1 - 8$$
$$\frac{7x}{7} = \frac{-7}{7}$$
$$x = -1$$

Substitute −1 for x in the equation $5x + 2y = 1$, and solve for y.

$$5x + 2y = 1$$
$$5(-1) + 2y = 1$$
$$-5 + 5 + 2y = 1 + 5$$
$$\frac{2y}{2} = \frac{6}{2}$$
$$y = 3$$

The solution of the system is $x = -1$ and $y = 3$.

Rubric

1 point for accurate work; 1 point for solution

7. There is no solution because the expression $-2x + 4y$ cannot equal both 5 and 6 at the same time for any values of x and y.

Rubric

1 point for answer; 1 point for explanation

8. Setting $3.5d + 2$ equal to $3d + 5$ and solving gives $d = 6$. Substituting 6 for d in $3.5d + 2$ gives $c = 23$.

The cost of each taxi is \$23 when the taxis travel 6 miles.

Rubric

1 point for distance; 1 point for cost

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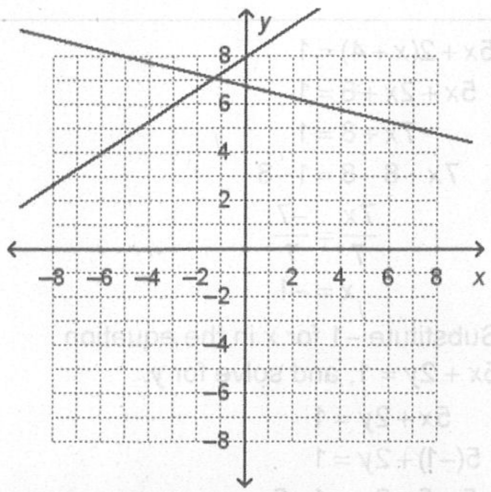
9.



No; although Dylan's estimate does appear to give the closest point with integer coordinates to the intersection point of the lines, it does not appear to be the best estimate of the coordinates to the nearest 0.5. To the nearest 0.5, the best estimate appears to be (−1.5, 7).

Rubric

0.5 point for each line; 1 point for answering no; 1 point for explanation with correct best estimate

10. a. Substitute $12t$ for d in the equation $d = 8(t + 2)$. Then solve for t.

$$12t = 8(t+2)$$
$$12t = 8t + 16$$
$$12t - 8t = 8t - 8t + 16$$
$$\frac{4t}{4} = \frac{16}{4}$$
$$t = 4$$

Substitute 4 for t in the equation $d = 12t$ and solve for d.

$$d = 12(4)$$
$$= 48$$

The solution is $t = 4$ and $d = 48$.

b. The solution $t = 4$ and $d = 48$ means that both boats are 48 miles from the marina 4 hours after the second boat left the marina.

Rubric

a. 1 point for solution; 1 point for solving system of equations algebraically

b. 1 point for the meaning of the time; 1 point for the meaning of the distance

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8.EE.8c Answers

1. D
2. D
3. D
4. C
5. C
6. E
7. B
8. F
9. No, because the equation of the line that passes through the points (0, 12) and (7, 10) in slope-intercept form is $y = -\frac{2}{7}x + 12$, and the equation of the line $2x + 7y = 21$ in slope-intercept form is $y = -\frac{2}{7}x + 3$. There is no solution because the lines have the same slope and different *y*-intercepts.

   Rubric

   0.5 point for slope-intercept form for the line that passes through the points (0, 12) and (7, 10); 0.5 point for slope-intercept form for the line $2x + 7y = 21$; 1 point for answer of "No solution"; 1 point for explanation
10. Jesse's store sold 35 pairs of skis and 48 snowboards in November.

    Rubric

    1 point for the number of pairs of skis; 1 point for the number of snowboards
11. Neither; the total weights of the monitors and printers are the same. The system $\begin{cases} m + p = 60 \\ 35m + 25p = 1750 \end{cases}$, where m is the number of monitors and p is the number of printers, describes this situation. The system has the solution $m = 25$ and $p = 35$. So, the monitors weigh a total of 25(35) = 875 pounds, and the printers weigh a total of 35(25) = 875 pounds.

    Rubric

    1 point for answer; 1 point for explanation
12. a. The equation for the first container is $w = 3.5t + 8$. The equation for the second container is $w = 3.25t + 24$.

    b. Substitute $3.5t + 8$ for w in the equation $w = 3.25t + 24$.

    $$3.5t + 8 = 3.25t + 24$$
    $$3.5t - 3.25t + 8 = 3.25t - 3.25t + 24$$
    $$0.25t + 8 - 8 = 24 - 8$$
    $$\frac{0.25t}{0.25} = \frac{16}{0.25}$$
    $$t = 64$$

    Substitute 64 for t in the equation $w = 3.5t + 8$.

    $w = 3.5(64) + 8 = 232$

    c. It would take 64 minutes for both to have 232 gallons of water.

    Rubric

    a. 0.5 point for each equation

    b. 1 point for solution; 1 point for solving system of equations algebraically

    c. 1 point
13. a. The equation for the speeding car is $d = 130t + 3150$. The equation for Marie's car is $d = 145t + 1725$.

    b. Substitute $130t + 3150$ for d in the equation $d = 145t + 1725$.

    $$130t + 3150 = 145t + 1725$$
    $$130t - 130t + 3150 = 145t - 130t + 1725$$
    $$3150 - 1725 = 15t + 1725 - 1725$$
    $$\frac{1425}{15} = \frac{15t}{15}$$
    $$95 = t$$

    Substitute 95 for t in the equation $d = 145t + 1725$.

    $d = 145(95) + 1725 = 15{,}500$

    c. The cars are 15,500 feet from the start of the chase 95 seconds after Marie's car reaches 145 feet per second.

    Rubric

    a. 0.5 point for each equation

    b. 1 point for solution; 1 point for showing work

    c. 1 point

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8.F.1 Answers

1. C
2. A
3. A
4. C, D, F
5. Yes, because the only requirement for a function is that each input has exactly one output assigned to it. An output can be assigned to any number of inputs.

   Rubric

   1 point for answer; 2 points for explanation

6. a. Yen is correct in saying that the set does not represent a function.

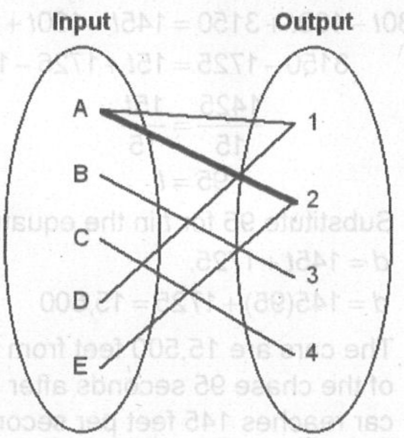
   b. Yen's reasoning is incorrect because it does not matter if different inputs have the same output assigned to them. The set does not represent a function because the ordered pairs (0, 0) and (0, 1) have the same *x*-value but different *y*-values.

   Rubric

   a. 1 point

   b. 1 point for stating Yen's reasoning is incorrect; 1 point for explanation of Yen's incorrect reasoning; 1 point for correct reasoning

7. Possible answer:



   The mapping diagram no longer represents a function because the input value A is now mapped to two different output values, 1 and 2.

   Rubric

   1 point for a correct segment; 2 points for explanation

8. The horizontal line does represent a function because every point on the line has a *y*-coordinate of 2. Although each input has the same output, there is only one output assigned to each input, as required by the definition of a function.

   The vertical line does not represent a function because every point on the line has an *x*-coordinate of 2. This means that an input of 2 has multiple outputs (in fact, infinitely many outputs) assigned to it, which violates the definition of a function.

   Rubric

   1 point each for indicating which line represents a function and which does not; 1 point each for giving a correct explanation about each line

9. a. The set of data must represent a function because the snake can have only one length at any given moment during its life. In other words, exactly one value of ℓ is paired with each value of *a*.

   b. The set of data does not necessarily represent a function because there could be two snakes that are the same age but have different lengths. In other words, more than one value of ℓ may be paired with a value of *a*.

   Rubric

   a. 1 point for recognizing the necessity of a function; 1 point for explanation

   b. 1 point for recognizing the possibility of not a function; 1 point for explanation

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8.F.2 Answers

1. A
2. D
3. B
4. B
5. A, C, D, F
6. a. Howard runs faster. Howard's speed is 5 miles per hour, which is faster than Martha's speed of 4.5 miles per hour.
   b. Martha has run $5 \cdot 4.5 = 22.5$ miles, and Howard has run $5 \cdot 5 = 25$ miles.

   Rubric

   a. 1 point for answer; 1 point for explanation
   b. 0.5 point for each distance
7. The ivy on side A reaches a greater height. The function giving the height of the ivy on side A is $h = 2t$. The function giving the height of the ivy on side B is $h = t + 10$. After 11 weeks, the height of the ivy on side A is $2 \cdot 11 = 22$ inches, while the height of the ivy on side B is $11 + 10 = 21$ inches.

   Rubric

   1 point for answer; 1 point for explanation
8. a. At time $t = 0$, barrel A holds 72 liters while barrel B holds 80 liters. So, barrel B initially has more water.
   b. Barrel B is emptied at the greater rate. From $w = -12t + 72$, the rate of change is -12, so barrel A loses 12 liters of water per minute. Barrel B loses 80 liters in 5 minutes, which is a rate of 16 liters per minute.
   c. Barrel B will be empty first. Emptying barrel A will take $\frac{72}{12} = 6$ minutes, while emptying barrel B will take 5 minutes.

   Rubric

   a. 1 point for answer; 1 point for justification
   b. 1 point for answer; 1 point for justification
   c. 1 point for answer; 1 point for justification
9. a. Eli begins 6 miles from the start of the trail, and that distance is decreasing at a rate of 1.65 miles per hour. So, the function $d = -1.65t + 6$ represents Eli's distance from the start of the trail on his way down the mountain.
   b. Set the two distances equal to obtain the equation $1.35t = -1.65t + 6$. Solving gives $t = 2$, which means that Eli and Alayna are at the same point on the trail in 2 hours. At that time, Alayna is $1.35 \cdot 2 = 2.7$ miles from the start of the trail.

   Rubric

   a. 1 point
   b. 1 point for answer; 1 point for explanation

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8.F.3 Answers

1. B
2. C
3. A
4. B, C, E
5. a. Yes
   b. No
   c. Yes
   d. Yes
   e. No
   f. No
6. No, the ordered pairs do not represent a linear function because graphing the ordered pairs shows that they do not lie on a single nonvertical line.

   Rubric

   1 point for answer; 1 point for explanation
7. No, Oliver is not correct. The equation $y = 1$ represents a linear function, but the equation $x = 1$ does not. The equation $y = 1$ is in the form $y = mx + b$, and its graph is a straight nonvertical line. However, the equation $x = 1$ cannot be put in the form $y = mx + b$, and its graph, although a line, is vertical.

   Rubric

   1 point for saying $y = 1$ is a linear function; 1 point for saying $x = 1$ is not a linear function; 2 points for explanation
8. The slope between any two of the points is always −2, so the points lie on the graph of a linear function. Since the graph has a y-intercept of 1, an equation of the linear function is $y = -2x + 1$.

   Rubric

   1 point for recognizing a linear function;
   1 point for equation
9. a. $y = 3x$

   b. $y = \frac{1}{11}x + \frac{2}{11}$

   c. The equation cannot be written in slope-intercept form.

   d. The equation cannot be written in slope-intercept form.

   e. The equations in parts a and b represent linear functions because the equations can be written in the form $y = mx + b$.

   Rubric

   a. 0.5 point
   b. 0.5 point
   c. 1 point
   d. 1 point
   e. 1 point for answer; 1 point for explanation
10. a. The equation representing the distance Vladimir walks over t hours is $d = 4t$. The equation representing the distance Cheryl walks over t hours is $d = 3t$.

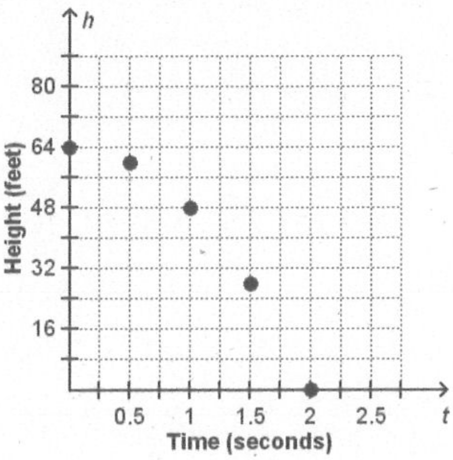
    b. The equations represent linear functions because they are in the form $y = mx + b$ (where b happens to be 0).

    Rubric

    a. 1 point for each equation
    b. 1 point for answer; 1 point for explanation
11. a.

t	0	0.5	1	1.5	2
h	64	60	48	28	0

b.



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c. No, the equation does not represent a linear function. Calculating the slope between the first two points gives $\frac{60-64}{0.5-0} = \frac{-4}{0.5} = -8$. Calculating the slope between the second and third points gives $\frac{48-60}{1-0.5} = \frac{-12}{0.5} = -24$.

Since the slope is not constant, the points do not lie on a line.

Rubric

a. 1 point

b. 1 point

c. 1 point for answer; 1 point for explanation

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8.F.4 Answers

1. C
2. B
3. A
4. D
5. B
6. C
7. E
8. D
9. a. $m = 24t + 36$

   b. A rate of change of 24 means that Julie makes 24 muffins every hour. An initial value of 36 means that Julie starts with 36 muffins, which is the number of muffins Tim made.

   Rubric

   a. 1 point

   b. 1 point for meaning of rate of change; 1 point for meaning of initial value

10. a. Calculate the rate of change using the first two ordered pairs in the table.

    $$\frac{1090-1150}{10-5} = \frac{-60}{5} = -12$$

    So, the hot air balloon's rate of change is –12 feet per second.

    b. In the table, consecutive time values differ by 5, while consecutive height values differ by –60. This means that the rate of change is constant.

    c. To find the height at the time the descent began, use the general linear function $h = mt + b$, substituting –12 for m and using an ordered pair from the table, such as $(t, h) = (5, 1150)$.

    $$h = mt + b$$
    $$1150 = -12(5) + b$$
    $$1150 = -60 + b$$
    $$1210 = b$$

    So, the hot air balloon's height at the time the descent began was 1210 feet.

    d. The function is $h = -12t + 1210$.

    Rubric

    1 point for each part

11. a. Jamal switched the two variables, p and g, when finding the rate of change and initial value. The correct rate of change is 20, and the correct initial value is –440. So, the function that models Jamal's profit is $p = 20g - 440$.

    Rate of change:

    $$\frac{-360-(-400)}{4-2} = \frac{40}{2} = 20$$

    Initial value: $p = mg + b$

    $$-400 = 20(2) + b$$
    $$-400 = 40 + b$$
    $$-440 = b$$

    b. A rate of change of 20 is the price at which he sells each game. Because each game sold adds \$20 to his income, his profit increases by \$20 each time a game is sold. An initial value of –\$440 represents the amount that Jamal paid for all the games in his store's inventory. It is negative because it's an expense to Jamal. Initially, Jamal has no income from the games, only the expense of purchasing them from his supplier.

    c. Jamal breaks even when $p = 0$.

    $$0 = 20g - 440$$
    $$440 = 20g$$
    $$22 = g$$

    So, Jamal will need to sell 22 games to break even.

    Rubric

    a. 1 point for identifying error; 1 point for correcting error; 1 point for reasonable work finding rate of change; 1 point for reasonable work finding initial value; 1 point for correct equation

    b. 0.5 point for interpretation of rate of change; 0.5 point for interpretation of initial value

    c. 1 point for answer; 1 point for reasonable work and explanation

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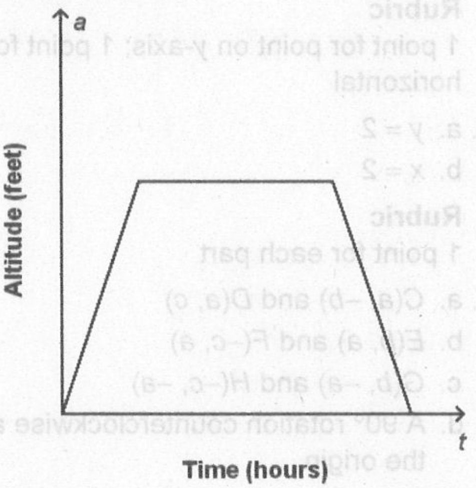
8.F.5 Answers

1. A
2. B
3. C
4. C, E
5. The height of the plant increases quickly at first, but the rate of increase slows down over time. The relationship is nonlinear.

   Rubric

   1 point for description; 1 point for nonlinear

6. Possible answer:



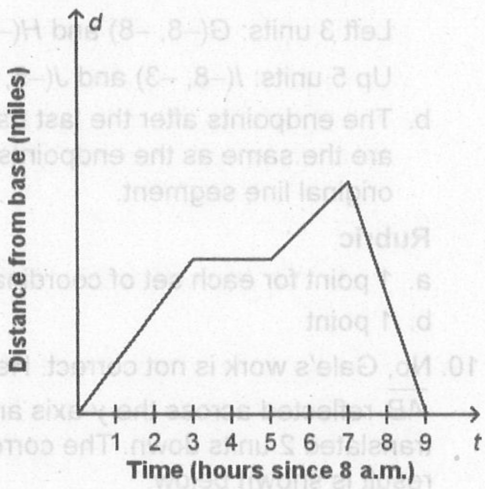
Rubric

1 point for each segment on graph

7. a. The slope is positive between 8 a.m. and 11 a.m. and between 1 p.m. and 3 p.m. The positive slope indicates that Oliver's distance from the base of the mountain is increasing.

   b. The slope is zero between 11 a.m. and 1 p.m. This means that Oliver's distance from the base is neither increasing nor decreasing. Since the trail is straight, this means that Oliver has stopped.

   c.



Oliver hiked faster overall on his way down the mountain than on his way up. On his way up, Oliver hiked for 5 hours, and on his way back he hiked for 2 hours. Since the distance covered was the same each way, his average rate of change, or average hiking speed, was faster on the way back.

Rubric

a. 1 point

b. 1 point

c. 1 point for drawing line segment; 1 point for answer; 1 point for explanation

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8.G.1a Answers

1. B
2. C
3. D
4. C
5. D
6. H
7. B
8. F
9. a. Right 3 units: $C(-5, -3)$ and $D(-2, -3)$

   Down 5 units: $E(-5, -8)$ and $F(-2, -8)$

   Left 3 units: $G(-8, -8)$ and $H(-5, -8)$

   Up 5 units: $I(-8, -3)$ and $J(-5, -3)$

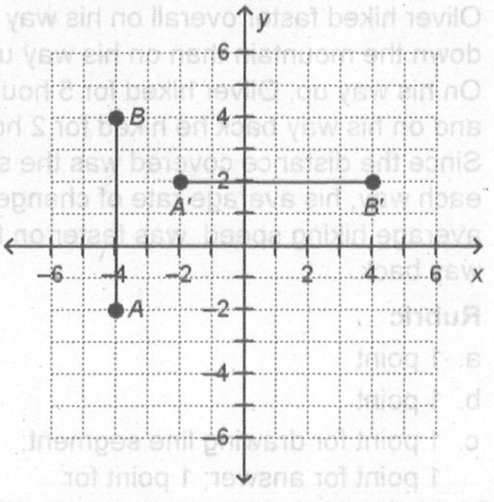
   b. The endpoints after the last translation are the same as the endpoints of the original line segment.

   Rubric

   a. 1 point for each set of coordinates

   b. 1 point

10. No, Gale's work is not correct. He drew $\overline{AB}$ reflected across the *y*-axis and translated 2 units down. The correct result is shown below.



The image segment is horizontal, while the original segment is vertical. The length of each line segment is the same, 6 units.

Rubric

1 point for answer; 1 point for saying Gale drew a reflection instead of a rotation; 1 point for correctly drawing the transformed segment; 1 point for saying the image segment is horizontal and the lengths are the same

11. The image of $\overleftrightarrow{AB}$ when reflected across the *x*-axis will pass through the point $(0, -3)$. The image, like $\overleftrightarrow{AB}$, is a horizontal line.

Rubric

1 point for point on *y*-axis; 1 point for horizontal

12. a. $y = 2$

    b. $x = 2$

Rubric

1 point for each part

13. a. $C(a, -b)$ and $D(a, c)$

    b. $E(b, a)$ and $F(-c, a)$

    c. $G(b, -a)$ and $H(-c, -a)$

    d. A 90° rotation counterclockwise about the origin

Rubric

a. 1 point for each endpoint

b. 1 point for each endpoint

c. 1 point for each endpoint

d. 2 points for the rotation

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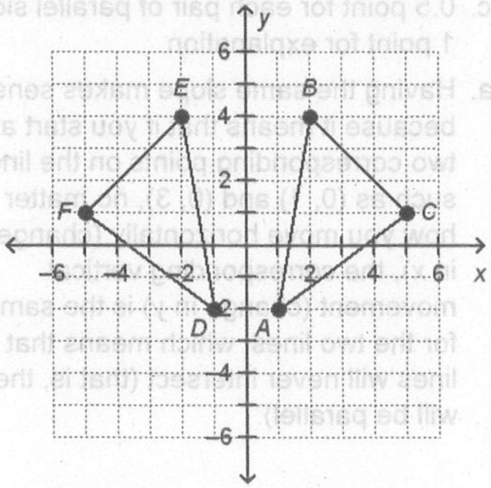
8.G.1b Answers

1. A
2. B
3. A
4. A
5. B, C, E, G
6. a. Different
   b. Same
   c. Different
   d. Different
   e. Same
   f. Same
7. $\angle F$ has the same measure as $\angle B$.

   $\angle G$ has the same measure as $\angle C$.

   Since a reflection across a line does not change the measures of angles, the measures of the angles in trapezoid *EFGH* are equal to the measures of the corresponding angles in the trapezoid *ABCD*.

   Rubric
   1 point for each answer; 1 point for explanation

8. 

   $m\angle D = m\angle A$; $m\angle E = m\angle B$; $m\angle F = m\angle C$

   Rubric
   1 point for image; 2 points for the equations

9. $m\angle A' = 52°$, $m\angle B' = 90°$, and $m\angle C' = 38°$. This illustrates the principle that the image of an angle after a translation is an angle having the same measure.

   Rubric
   1 point for each angle measure; 1 point for stating general principle

10. The measures of the angles do not change under any of the three transformations; the measure of each angle remains 90°. This situation illustrates the principle that the measure of an angle does not change when the angle is rotated, reflected, or translated.

    Rubric
    1 point for stating the angle does not change under any transformation;
    1 point for general principle

11. $\Delta C'AC$ is an isosceles triangle because the image of a line segment after a reflection is a line segment of the same length. So, $AC = AC'$. Because $\Delta C'AC$ is a triangle with two sides of equal length, it is an isosceles triangle.

    $\angle C'$ has the same measure as $\angle C$ because the image of an angle after a reflection is an angle of the same measure.

    The measure of $\angle C'AC$ is twice the measure of $\angle BAC$. $\angle BAC'$ has the same measure as $\angle BAC$ because the image of an angle after a reflection is an angle of the same measure. Notice that $\angle C'AC$ is composed of $\angle BAC$ and $\angle BAC'$. Substitute $m\angle BAC$ for $m\angle BAC'$ in the equation $m\angle C'AC = m\angle BAC' + m\angle BAC$ and simplify the right side.

    $$\begin{aligned} m\angle C'AC &= m\angle BAC' + m\angle BAC \\ &= m\angle BAC + m\angle BAC \\ &= 2 \cdot m\angle BAC \end{aligned}$$

    Rubric
    1 point for the type of triangle; 1 point for each angle comparison; 2 points for explanation

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8.G.1c Answers

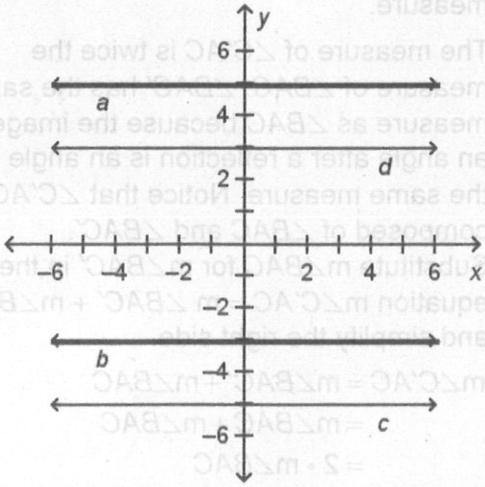
1. C
2. D
3. A
4. B
5. C, F
6. In rhombus *ABCD*, sides $\overline{AB}$ and $\overline{CD}$ are parallel, and sides $\overline{AD}$ and $\overline{BC}$ are parallel. The transformation is a reflection across the *x*-axis. Because the images of parallel lines after a reflection are parallel lines, the images of $\overline{AB}$ and $\overline{CD}$ are still parallel, and the images of $\overline{AD}$ and $\overline{BC}$ are still parallel.

   Rubric
   0.5 point for each pair of parallel lines;
   1 point for saying images are parallel;
   1 point for explanation

7. The two shelves would still be parallel if they were moved up 2 inches or down 2 inches. This illustrates that vertical translations map one set of parallel lines to another set of parallel lines.

   Rubric
   1 point for answering each question

8. Line *c* is the image of line *a*. Line *d* is the image of line *b*.

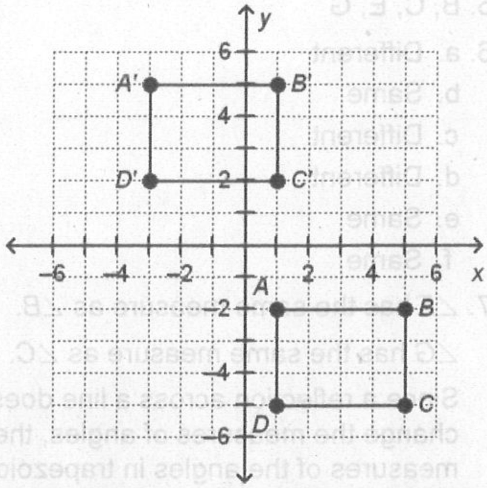


The images of lines *a* and *b*, lines *c* and *d*, respectively, are also horizontal and parallel.

Rubric
1 point for each reflection; 1 point for stating the images are horizontal;
1 point for stating the images are parallel

9. a. $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$

   b.



   c. $\overline{A'B'} \parallel \overline{C'D'}$ and $\overline{A'D'} \parallel \overline{B'C'}$ because translations of figures preserve parallel sides.

   Rubric
   a. 0.5 point for each pair of parallel sides
   b. 1 point
   c. 0.5 point for each pair of parallel sides; 1 point for explanation

10. a. Having the same slope makes sense because it means that if you start at two corresponding points on the lines, such as (0, 1) and (0, 3), no matter how you move horizontally (change in *x*), the corresponding vertical movement (change in *y*) is the same for the two lines, which means that the lines will never intersect (that is, they will be parallel).

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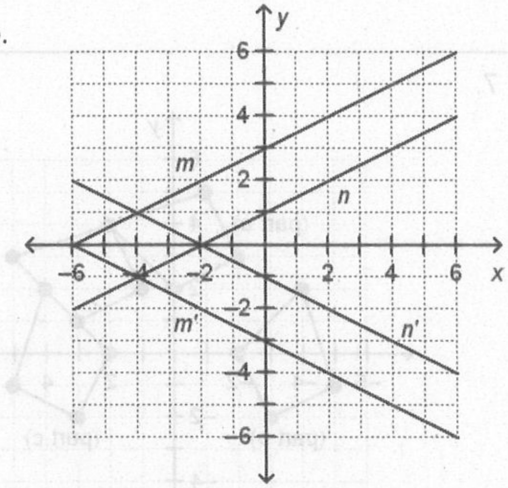
b.



c. Line *m′* has the equation $y = -\frac{1}{2}x - 3$.

Line *n′* has the equation $y = -\frac{1}{2}x - 1$.

d. Yes, the lines are parallel because the slope of both lines is $-\frac{1}{2}$.

Rubric

a. 1 point
b. 1 point
c. 0.5 point for each equation
d. 1 point for answer; 1 point for explanation

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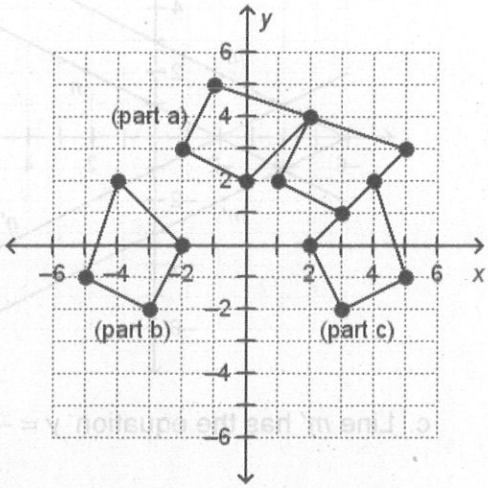
8.G.2 Answers

1. B
2. B
3. A, D
4. Yes, the triangles are congruent because there exists a sequence of translations, reflections, and/or rotations that maps $\triangle ABC$ to $\triangle DEF$. One such sequence is to rotate $\triangle ABC$ 90° clockwise about the origin and then translate the image down 2 units. The final image is $\triangle DEF$.

   Rubric
   1 point for answer; 1 point for sequence of transformations
5. The vertices of the given figure are (–1, 2), (–2, 5), (–4, 5), (–5, 2).
   a. These vertices describe the image of the given figure after a translation 6 units right and 4 units down. So, the figure is congruent to the given figure.
   b. Possible answer: A translation 5 units to the right and 1 unit down will map (–1, 2) to (4, 1), (–2, 5) to (3, 4), and (–4, 5) to (1, 4). However, the same translation does not map (–5, 2) to (0, 3). So, the figure is not congruent to the given figure.

   Rubric
   a. 1 point for answer; 1 point for explanation using transformations
   b. 1 point for answer; 1 point for explanation using transformations
6. No, *ABCD* is not congruent to *WZYZ*. Reflecting *ABCD* across the *y*-axis and translating 3 units up maps *B* to *X*, *C* to *Y*, and *D* to *Z* but does not map *A* to *W*. So, the figures are not congruent.

   Rubric
   1 point for saying not congruent;
   2 points for explanation
7.



Yes, the original figure and the final image are congruent. The final image is the result of a translation, a rotation, and a reflection.

Rubric
1 point for each transformation;
1 point for saying congruent;
1 point for explanation

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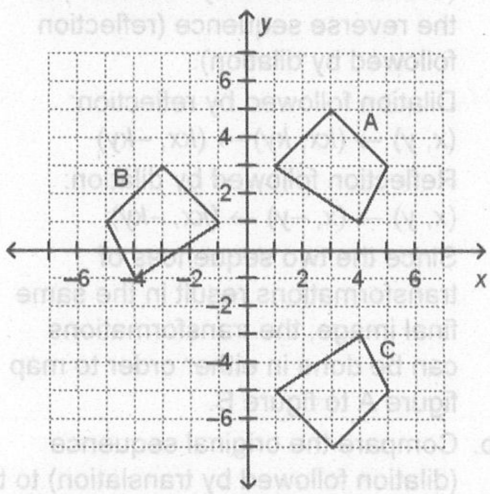
8.G.3 Answers

1. A
2. C
3. A
4. A, B, D, F
5. a. (3, 6), (5, 5), and (5, 6)
   b. $(x, y) \rightarrow (x + 2, y + 1)$
   c. (5, 6), (5, 7), (7, 6), and (7, 7)

   Rubric

   1 point for each part
6. No, Mila is not correct. When reflecting across the *x*-axis, the *y*-coordinate changes sign, not the *x*-coordinate. So, the correct rule is $(x, y) \rightarrow (x, -y - 2)$. The correct image of figure A is figure C in the coordinate plane below.

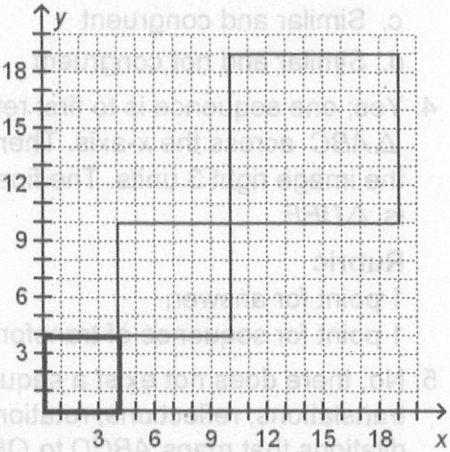


   Rubric

   1 point for identifying error;
   1 point for correct transformation rule;
   1 point for explanation;
   1 point for correct image

7. a. Dilation $(x, y) \rightarrow \left(\frac{3}{2}x, \frac{3}{2}y\right)$ and translation $(x, y) \rightarrow (x+4, y+4)$

   b.



   c. Under the dilation $(x, y) \rightarrow \left(\frac{3}{2}x, \frac{3}{2}y\right)$, the sides of each image are $\frac{3}{2}$ times longer than the sides of its preimage.

$$\left(\frac{3}{2}s\right)^2 = \left(\frac{3}{2}\right)^2 \cdot s^2$$
$$= \frac{9}{4}s^2$$

   So, the area of each image is $\frac{9}{4}$ times the area of its preimage.

   Rubric

   a. 1 point for each transformation
   b. 1 point for each graph
   c. 1 point for answer; 2 points for explanation

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8.G.4 Answers

1. B
2. C
3. a. Similar and congruent
   b. Similar and not congruent
   c. Similar and congruent
   d. Similar and not congruent
4. Yes; one sequence is to first reflect $\triangle ABC$ across the x-axis. Then translate the image right 3 units. The final image is $\triangle DEF$.

   Rubric
   1 point for answer;
   1 point for sequence of transformations
5. No, there does not exist a sequence of translations, reflections, rotations, and/or dilations that maps $ABCD$ to $QRST$. In $ABCD$, the lengths of the parallel, and opposite, sides $\overline{AB}$ and $\overline{CD}$ are each 3 units, and these sides are 6 units apart (because they lie on the vertical lines $x = -9$ and $x = -3$). In $QRST$, the lengths of the parallel, and opposite, sides $\overline{QR}$ and $\overline{ST}$ are each 1 unit, and these sides are 3 units apart (because they lie on the horizontal lines $y = 4$ and $y = 1$). So, the ratio of AB to QR is 3:1, but the ratio of the distances between these sides and their opposite sides is 2:1. So, no dilation exists that will map the corresponding lengths and the corresponding distances because different scale factors are needed.

   Rubric
   1 point for answer; 2 points for explanation
6. Callie is correct in saying the figures are not congruent. There exists no sequence of translations, reflections, and/or rotations that will map figure 1 to figure 2.

   Callie is incorrect in saying the figures are not similar. One sequence of transformations that maps figure 1 to figure 2 is the dilation $(x, y) \rightarrow \left(\frac{2}{3}x, \frac{2}{3}y\right)$ followed by the rotation $(x, y) \rightarrow (y, -x)$. Since there exists a sequence of translations, reflections, rotations, and/or dilations, the two figures are similar.

   Rubric
   1 point for saying not congruent is incorrect; 1 point for explanation; 1 point for saying similar is correct; 1 point for each sequence in transformation
7. a. Compare the original sequence (dilation followed by reflection) to the reverse sequence (reflection followed by dilation).
   Dilation followed by reflection:
   $(x, y) \rightarrow (kx, ky) \rightarrow (kx, -ky)$
   Reflection followed by dilation:
   $(x, y) \rightarrow (x, -y) \rightarrow (kx, -ky)$
   Since the two sequences of transformations result in the same final image, the transformations can be done in either order to map figure A to figure B.

   b. Compare the original sequence (dilation followed by translation) to the reverse sequence (translation followed by dilation).
   Dilation followed by translation:
   $(x, y) \rightarrow (kx, ky) \rightarrow (kx + a, ky)$
   Translation followed by dilation:
   $(x, y) \rightarrow (x + a, y) \rightarrow (k(x + a), ky)$
   Since $k(x + a) = kx + ka \neq kx + a$ (except in the special cases where $k = 1$ or $a = 0$), the two sequences of transformations result in different final images. So, the transformation must be done in the particular order given to map figure C to figure D.

   Rubric
   a. 2 points for proof; 1 point for result
   b. 2 points for proof; 1 point for result

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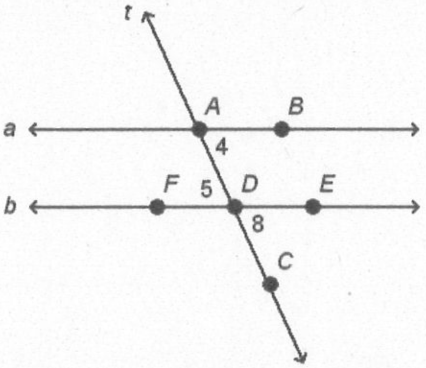
8.G.5 Answers

1. A
2. C
3. B
4. B
5. A, E, F
6. Let the sides of $\angle 4$ be $\overrightarrow{AB}$ and $\overrightarrow{AC}$, let the sides of $\angle 8$ be $\overrightarrow{DE}$ and $\overrightarrow{DC}$, and let the sides of $\angle 5$ be $\overrightarrow{DF}$ and $\overrightarrow{DA}$.



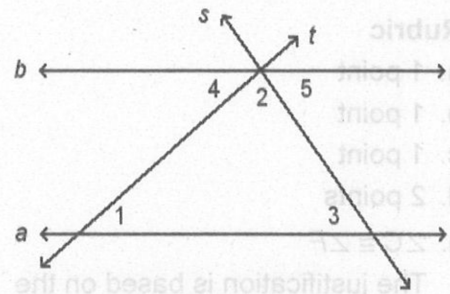
Translate the vertex of $\angle 4$ (point A) to the vertex of $\angle 8$ (point D). The same translation maps $\overrightarrow{AB}$ to $\overrightarrow{DE}$, because translations always map a line to a parallel line. Similarly, $\overrightarrow{AC}$ is mapped to $\overrightarrow{DC}$, because the translation is along the line t. Since the rays that form $\angle 4$ are mapped to the rays that form $\angle 8$, $\angle 4 \cong \angle 8$.

Rotate $\angle 8$ 180° about its vertex (point D). $\overrightarrow{DE}$ and $\overrightarrow{DC}$ are mapped to their opposite rays $\overrightarrow{DF}$ and $\overrightarrow{DA}$, respectively. $\overrightarrow{DF}$ and $\overrightarrow{DA}$ are the sides of $\angle 5$. So, $\angle 5 \cong \angle 8$, and $\angle 4 \cong \angle 5$ by transitivity.

Rubric

2 points for showing $\angle 4 \cong \angle 8$; 2 points for showing $\angle 8 \cong \angle 5$; 1 point for concluding that $\angle 4 \cong \angle 5$

7.



a. Lines a and b are shown in the figure above.

b. Lines s and t are shown in the figure above; transversals

c. $\angle 4$; $\angle 5$; congruent

d. $\angle 4$; $\angle 2$; $\angle 5$; 180°
(Any order for the angles is correct.)

e. Since $m\angle 4 = m\angle 1$ and $m\angle 5 = m\angle 3$, the sum of the measures of the angles of a triangle is equal to the sum of the measures of the three angles, $\angle 4$, $\angle 2$, and $\angle 5$, that form a straight angle along line b, and that sum is 180°.

Rubric

a. 0.5 point

b. 0.5 point

c. 1 point for each set of angles; 0.5 point for congruent

d. 0.5 point for angles; 1 point for 180°

e. 1 point for answer; 1 point for explanation

8. a. The exterior angle is $\angle 4$. The remote interior angles are $\angle 1$ and $\angle 2$.

b. $m\angle 1$; $m\angle 2$; $m\angle 3$

c. $\angle 3$ and $\angle 4$ are a linear pair, so $m\angle 3 + m\angle 4 = 180°$.

d. The measure of an exterior angle is equal to the sum of its remote interior angles.

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$$\begin{aligned} m\angle 1 + m\angle 2 + m\angle 3 &= m\angle 3 + m\angle 4 \\ -m\angle 3 &= -m\angle 3 \\ \hline m\angle 1 + m\angle 2 &= m\angle 4 \end{aligned}$$

Rubric

a. 1 point
b. 1 point
c. 1 point
d. 2 points

9. a. $\angle C \cong \angle F$

The justification is based on the triangle sum theorem and substitution:

$$\begin{aligned} m\angle C &= 180° - m\angle A - m\angle B \\ &= 180° - m\angle D - m\angle E \\ &= m\angle F \end{aligned}$$

b. Yes, $\triangle ABC$ and $\triangle DEF$ are similar. All three angles in $\triangle ABC$ are congruent to the corresponding angles in $\triangle DEF$. The angles of a triangle determine its shape, and two figures that have the same shape are similar.

c. There is not enough information to conclude that $\triangle ABC$ and $\triangle DEF$ are congruent, because no information is given about the lengths of the sides of the triangles. The lengths of the sides of a triangle determine its size, and congruent figures must not only have the same shape but also the same size.

Rubric

a. 1 point for answer;
1 point for explanation
b. 1 point for concluding similarity;
1 point for not being able to conclude congruency; 0.5 point for each explanation

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8.G.6 Answers

1. B
2. D
3. C
4. a. The length of each side of the larger square is $a + b$. So, the area is $(a+b)^2$.

   b. The lengths of the legs of each right triangle serve as the base and height of the triangle when finding its area.

   So, the area of one triangle is $\frac{1}{2}ab$, and the combined area of all four triangles is $4\left(\frac{1}{2}ab\right) = 2ab$.

   The length of a side of the smaller square is c. So, the area of the smaller square is c^2, and the sum of the areas of the four triangles and the smaller square is $2ab + c^2$.

   c.
$$(a+b)^2 = 2ab + c^2$$
$$a^2 + b^2 + 2ab = 2ab + c^2$$
$$a^2 + b^2 + 2ab - 2ab = 2ab - 2ab + c^2$$
$$a^2 + b^2 = c^2$$

Rubric

a. 1 point

b. 1 point for area of triangles; 1 point for area of square

c. 1 point

5. The area A of a trapezoid is given by the formula $A = \frac{1}{2}h(b_1 + b_2)$, where h is the height and b_1 and b_2 are the bases. This trapezoid has height $h = a + b$, a lower base $b_1 = a$, and an upper base $b_2 = b$.

$$A = \frac{1}{2}(a+b)(a+b)$$
$$= \frac{1}{2}\left(a^2 + b^2 + 2ab\right)$$
$$= \frac{1}{2}a^2 + \frac{1}{2}b^2 + ab$$

There are two triangles with base a and height b. The area of each of these triangles is $\frac{1}{2}ab$. The area of the triangle with base c and height c is $\frac{1}{2}c^2$. So, the sum of the areas of these three triangles is $2\left(\frac{1}{2}ab\right) + \frac{1}{2}c^2 = ab + \frac{1}{2}c^2$.

The area of the trapezoid is equal to the sum of the areas of the three triangles.

$$\frac{1}{2}a^2 + \frac{1}{2}b^2 + ab = ab + \frac{1}{2}c^2$$
$$\frac{1}{2}a^2 + \frac{1}{2}b^2 + ab - ab = ab - ab + \frac{1}{2}c^2$$
$$\frac{1}{2}a^2 + \frac{1}{2}b^2 = \frac{1}{2}c^2$$
$$a^2 + b^2 = c^2$$

Notice that the two congruent right triangles that are part of the trapezoid have sides of length a, b, and c. So, for a right triangle with legs of length a and b and a hypotenuse of length c, $a^2 + b^2 = c^2$.

Rubric

1 point for area of trapezoid; 1 point for area of triangle with sides a and b;
1 point for area of triangle with sides c;
1 point for finding sum of areas; 1 point for conclusion

6. a. By the Pythagorean theorem, $a^2 + b^2 = f^2$. So, the length of the hypotenuse of ΔDEF is $f = \sqrt{a^2 + b^2}$.

   b. It is given that for ΔABC, $a^2 + b^2 = c^2$. So, $c = \sqrt{a^2 + b^2}$. Since $f = \sqrt{a^2 + b^2}$ from part a, you know that $c = f$ by transitivity. So, the side of length c in ΔABC and the hypotenuse of ΔDEF are congruent.

   c. Yes, the triangles are congruent. The corresponding angles in both triangles are also congruent.

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d. You can conclude that $\triangle ABC$ is a right triangle. Since $\triangle ABC$ and $\triangle DEF$ are congruent triangles, the angle formed by the sides of length *a* and *b* in $\triangle ABC$ is a right angle because $\angle F$ is a right angle in $\triangle DEF$.

Rubric

a. 1 point

b. 1 point for answer;
1 point for explanation

c. 1 point for saying the triangles are congruent; 1 point for saying the corresponding angles are congruent

d. 1 point for conclusion;
1 point for explanation

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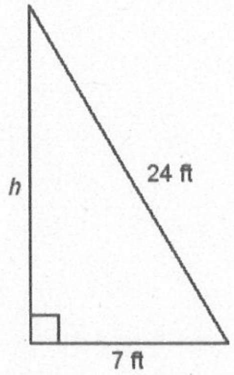
8.G.7 Answers

1. B
2. B
3. D
4. A, B, C, D
5. B, C, D, E
6.



By the Pythagorean theorem, $7^2 + h^2 = 24^2$, where h is the height of the rain gutters.

$$7^2 + h^2 = 24^2$$
$$49 - 49 + h^2 = 576 - 49$$
$$h^2 = 527$$
$$h = \sqrt{527}$$
$$h \approx 23$$

So, the rain gutters are approximately 23 ft from the ground.

Rubric
1 point for reasonable drawing; 1 point for height; 1 point for reasonable work

7. The sticks divide the kite into four right triangles. The lengths of the legs of the top two triangles are $\frac{80}{2} = 40$ cm and $90 - 60 = 30$ cm. By the Pythagorean theorem, $40^2 + 30^2 = x^2$, where x is the unknown side length for the top two triangles.

$$40^2 + 30^2 = x^2$$
$$1600 + 900 = x^2$$
$$2500 = x^2$$
$$50 = x$$

The lengths of the legs of the bottom two triangles are 40 cm and 60 cm. By the Pythagorean theorem, $40^2 + 60^2 = y^2$, where y is the unknown side length.

$$40^2 + 60^2 = y^2$$
$$1600 + 3600 = y^2$$
$$5200 = y^2$$
$$\sqrt{5200} = y$$
$$72 \approx y$$

So, the perimeter of the kite is about 50 cm + 50 cm + 72 cm + 72 cm = 244 cm. So, Manuel will need about 244 cm of material for the border.

Rubric
1 point for answer; 2 points for reasonable work

8. First, find the length of the unknown side. By the Pythagorean theorem, $BC^2 + 8^2 = 9^2$.

$$BC^2 + 8^2 = 9^2$$
$$BC^2 + 64 - 64 = 81 - 64$$
$$BC^2 = 17$$
$$BC = \sqrt{17}$$
$$BC \approx 4.1$$

So, the length of the unknown side is about 4.1 cm.

Then, find the approximate area using the fact that the lengths of the legs in a right triangle are the base and height of the triangle.

$$\text{Area} = \frac{1}{2} \cdot 8 \cdot \sqrt{17} \approx 16.5$$

So, the approximate area of $\triangle ABC$ is 16.5 cm^2.

Rubric
1 point for length of unknown side;
1 point for area;
1 point for reasonable work

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9. a. Leo is correct. The top of the block is 5 in. by 4 in. By the Pythagorean theorem, $5^2 + 4^2 = AB^2$.

$$\begin{aligned} 5^2 + 4^2 &= AB^2 \\ 25 + 16 &= AB^2 \\ 41 &= AB^2 \\ \sqrt{41} &= AB \\ 6 &\approx AB \end{aligned}$$

So, the length of the cut from point A to point B, rounded to the nearest inch, is 6 in.

b. Leo is not correct. The length of a diagonal is the length of the hypotenuse of right $\triangle ABC$. For this triangle, you know that $AB^2 = 41$ from part a, and you know that $BC = 4$. By the Pythagorean theorem, $AB^2 + BC^2 = AC^2$.

$$\begin{aligned} 41 + 4^2 &= AC^2 \\ 41 + 16 &= AC^2 \\ 57 &= AC^2 \\ \sqrt{57} &= AC \\ 8 &\approx AC \end{aligned}$$

So, the length of a diagonal, rounded to the nearest inch, is 8 in.

Rubric

a. 1 point for saying Leo is correct; 1 point for explanation

b. 1 point for saying Leo is incorrect; 1 point for explanation; 1 point for correct length

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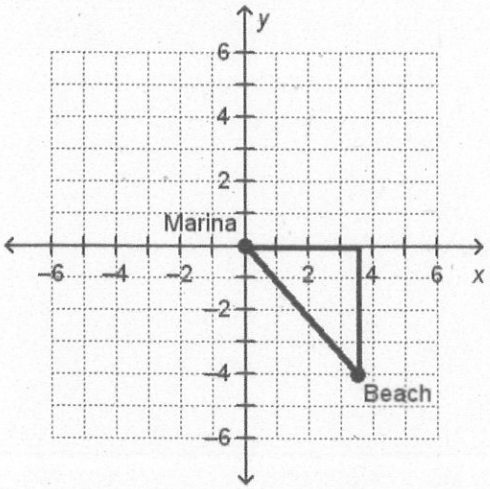
8.G.8 Answers

1. C
2. D
3. C
4. B
5. C
6. G
7. F
8. E
9. Since the marina is located at (0, 0) and the beach is 3.5 miles east and 4 miles south, the beach is located at $(0 + 3.5, 0 - 4) = (3.5, -4)$. Create a right triangle using the points (0, 0) and (3.5, −4) as shown.



The length of the horizontal leg of the triangle is $|0 - 3.5| = 3.5$ miles, and the length of the vertical leg is $|-4 - 0| =$ 4 miles. By the Pythagorean theorem, the distance between the marina and the beach is about 5.3 miles.

$$3.5^2 + 4^2 = c^2$$
$$12.25 + 16 = c^2$$
$$28.25 = c^2$$
$$\sqrt{28.25} = c$$
$$5.3 \approx c$$

Rubric
1 point for answer; 1 point for using coordinate plane; 1 point for explanation

10. The shape of the garden is a right triangle.

The vertical leg extends from (3, 2) to (3, −4). So, the length of this side of the garden is $|2 - (-4)| = 6$ feet.
The horizontal leg extends from (−4, 2) to (3, 2). So, the length of this side of the garden is $|-4 - 3| = 7$ feet.

By the Pythagorean theorem, the length of the remaining side of the garden is about 9.2 feet.

$$6^2 + 7^2 = c^2$$
$$36 + 49 = c^2$$
$$85 = c^2$$
$$\sqrt{85} = c$$
$$9.2 \approx c$$

So, Noah will need approximately $6 + 7 + 9.2 = 22.2$ feet of fence. Since Noah has only 22 feet of fencing, he does not have enough to enclose his garden.

Rubric
1 point for answer;
2 points for explanation

11. The shortest piece of wood Jeffrey can use will be the length of the longer side of the support that is shown in the diagram. The longer side is the line segment with endpoints (−3, −4) and (4, 4). This side forms a right triangle with the horizontal and vertical beams. The length of the vertical leg of the triangle is $|-4 - 4| =$ 8 inches. The length of the horizontal leg of the triangle is $|-3 - 4| = 7$ inches. By the Pythagorean theorem, the length of the longer side of the support is about 10.6 inches.

$$8^2 + 7^2 = c^2$$
$$64 + 49 = c^2$$
$$113 = c^2$$
$$\sqrt{113} = c$$
$$10.6 \approx c$$

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So, the shortest piece of wood Jeffrey can use to make the support is about 10.6 inches long.

Rubric

1 point for answer;
1 point for reasonable work

12. a. Since $A(x_1, y_1)$ and $C(x_2, y_1)$ have the same y-coordinate, $\overline{AC}$ is horizontal, and the length of $\overline{AC}$ is the absolute value of the difference between the x-coordinates, $|x_1 - x_2|$. Similarly, since $B(x_2, y_2)$ and $C(x_2, y_1)$ have the same x-coordinate, $\overline{BC}$ is vertical, and the length of $\overline{BC}$ is the absolute value of the difference between the y-coordinates, $|y_1 - y_2|$.

    b. $\triangle ABC$ is a right triangle with legs $\overline{AC}$ and $\overline{BC}$ and hypotenuse $\overline{AB}$. Substitute the lengths of the legs $\overline{AC}$ and $\overline{BC}$ into the Pythagorean theorem and solve for the length of $\overline{AB}$.

$$a^2 + b^2 = c^2$$

$$|x_1 - x_2|^2 + |y_1 - y_2|^2 = AB^2$$

$$\sqrt{|x_1 - x_2|^2 + |y_1 - y_2|^2} = AB$$

Rubric

a. 1 point for each length;
1 point for explanation

b. 1 point for answer;
1 point for reasonable work

c. 1 point for answer;
1 point for reasonable work

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8.G.9 Answers

1. B
2. C
3. B
4. B
5. D
6. A, B, D
7. The volume V of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$. The radius of the softball is 3.75 cm, and the radius of the table tennis ball is 2 cm. So, the ratio of the volumes of the softball and the table tennis ball is $\frac{\frac{4}{3}\pi \cdot 3.75^3}{\frac{4}{3}\pi \cdot 2^3}$. So, the volume of the softball is about 6.6 times the volume of the table tennis ball.

$$\frac{\frac{4}{3}\pi \cdot 3.75^3}{\frac{4}{3}\pi \cdot 2^3} = \frac{3.75^3}{2^3} = \frac{52.734375}{8} \approx 6.6$$

Rubric
1 point for ratio;
2 points for reasonable work

8. The trophy is made up of two shapes, a sphere for the basketball and a cylinder for the base.

The volume V of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere. The radius of the basketball is 2.35 in. So, the volume of the basketball is about 54.3 in^3.

$$\begin{aligned} V &\approx \frac{4}{3} \cdot 3.14 \cdot 2.35^3 \\ &= \frac{4}{3} \cdot 3.14 \cdot 12.977875 \\ &\approx 54.3 \end{aligned}$$

The volume V of a cylinder is given by the formula $V = \pi r^2 h$, where r is the radius and h is the height of the cylinder. Since the radius of the base is equal to the radius of the basketball, the radius is 2.35 in. The height of the base is 2 in. So, the volume of the base is about 34.7 in^3.

$$\begin{aligned} V &\approx 3.14 \cdot 2.35^2 \cdot 2 \\ &= 3.14 \cdot 5.5225 \cdot 2 \\ &\approx 34.7 \end{aligned}$$

So, the total volume of the trophy is about 54.3 in^3 + 34.7 in^3 = 89.0 in^3.

Rubric
1 point for answer; 3 points for reasonable work

9. The sharpened end of a round pencil is in the shape of a cone. The volume V of a cone is given by the formula $V = \frac{1}{3}\pi r^2 h$. The base diameter of the sharpened end of the pencil is 5 mm, so the base radius is $\frac{5}{2} = 2.5$ mm. The height is 15 mm. So, the volume is about 98.125 mm^3.

$$\begin{aligned} V &\approx \frac{1}{3} \cdot 3.14 \cdot 2.5^2 \cdot 15 \\ &= \frac{1}{3} \cdot 3.14 \cdot 6.25 \cdot 15 \\ &= 98.125 \end{aligned}$$

The rest of the pencil is in the shape of a cylinder. The volume V of a cylinder is given by the formula $V = \pi r^2 h$. The radius is 2.5 mm, and the height is 175 mm. So, the volume is about 3,434.375 mm^3.

$$\begin{aligned} V &\approx 3.14 \cdot 2.5^2 \cdot 175 \\ &= 3.14 \cdot 6.25 \cdot 175 \\ &= 3{,}434.375 \end{aligned}$$

So, the total volume of the pencil is about 98.125 mm^3 + 3,434.375 mm^3 = 3,532.5 mm^3.

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Rubric

1 point for answer; 3 points for reasonable work

10. The volume V of a cylinder is given by the formula $V = \pi r^2 h$. The radius of the candle is 2 in., and the height is 9 in. So, the volume of the candle is about 113.0 in^3.

$$V \approx 3.14 \cdot 2^2 \cdot 9$$
$$= 3.14 \cdot 4 \cdot 9$$
$$\approx 113.0$$

Rubric

1 point for answer;
1 point for reasonable work

11. a. Convert the base diameter of the cone's base to yards.

$$18 \text{ ft} \cdot \frac{1 \text{ yd}}{3 \text{ ft}} = 6 \text{ yd}$$

So, the cone and the cylinder have the same base diameter, which means that they have the same base radius.

b. The volume V of the cone is given by the formula $V = \frac{1}{3}\pi r^2 h_1$, where r is the base radius and h_1 is the height of the cone. The volume V of the cylinder is given by the formula $V = \pi r^2 h_2$, where r is the radius and h_2 is the height of the cylinder. So, if the cone is placed inside the cylinder, the remaining volume V of the cylinder is

$$V = \pi r^2 h_2 - \frac{1}{3}\pi r^2 h_1 = \pi r^2\left(h_2 - \frac{1}{3}h_1\right).$$

c. From part a, you know that the base radius for both the cone and cylinder is $\frac{6}{2} = 3$ yd. Convert the height of the cone to yards.

$$27 \text{ ft} \cdot \frac{1 \text{ yd}}{3 \text{ ft}} = 9 \text{ yd}$$

So, the height of the cone is 9 yd, while the height of the cylinder is 12 yd. Substitute 3 for r, 9 for h_1, and 12 for h_2 in the formula from part b.

$$V \approx 3.14 \cdot 3^2\left(12 - \frac{1}{3} \cdot 9\right)$$
$$= 3.14 \cdot 9\left(12 - \frac{1}{3} \cdot 9\right)$$
$$= 3.14 \cdot 9(12 - 3)$$
$$= 3.14 \cdot 9(9)$$
$$= 254.34$$

So, the remaining volume of the cylinder is about 254.34 yd^3. (Note: The alternative calculation using feet is equally valid and gives a remaining volume of about 6867.18 ft^3.)

d. If the cone and the cylinder have the same base radius and the same height, the formula for the remaining volume of the cylinder is $V = \pi\left(r^2 h - \frac{1}{3}r^2 h\right) = \pi r^2 h\left(1 - \frac{1}{3}\right) = \frac{2}{3}\pi r^2 h$. So, the remaining volume of the cylinder is $\frac{2}{3}$ of the original volume of the cylinder.

Rubric

a. 1 point

b. 2 points

c. 1 point for answer; 1 point for reasonable work

d. 1 point for formula; 1 point for saying remaining volume is $\frac{2}{3}$ of the original volume

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8.SP.1 Answers

1. A
2. C
3. C
4. B, E
5. a. There is a positive, linear association between the test scores for parts 1 and 2. There is a cluster of data points around (75, 75).

   b. The data points tend to follow the line $y = x$ where x is the part 1 score and y is the part 2 score. So, the students tended to do the same on both parts of the exam.

   Rubric

   a. 1 point for positive, linear association; 1 point for cluster

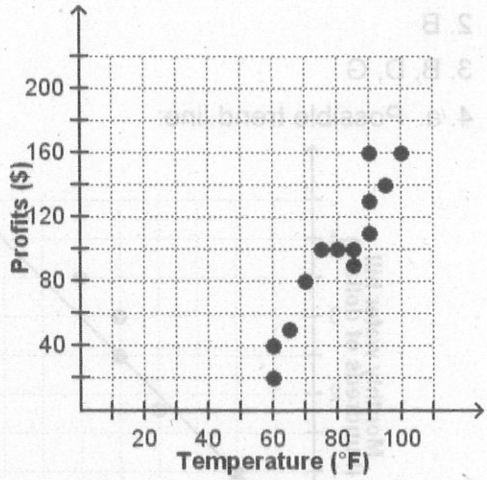
   b. 1 point for answer; 1 point for explanation

6. Sydney is incorrect in saying there is no association. As the number of people in a household increases, the amount spent on groceries increases. So, there is a positive association. The data points tend to follow a curve, so there is a nonlinear association between the two variables.

   Rubric

   1 point for answer; 2 points for explanation

7. a.



   b. There is a positive, linear association between profit and temperature. As the temperature increases, Calvin's profit tends to increase linearly.

   Rubric

   a. 1 point

   b. 1 point for association; 1 point for explanation

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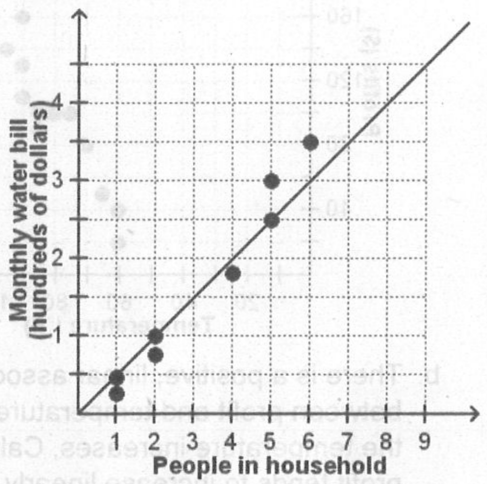
8.SP.2 Answers

1. A
2. B
3. B, D, G
4. a. Possible trend line:



There is a positive, linear association between the number of people in a household and the household's monthly water bill.

b. The trend line is shown in part a. The trend line is a good fit because all of the data points are close to the line and there are about an equal number of points above and below the line.

c. The trend line appears to pass through (3, 1.5), so for a water bill of $150, there are approximately 3 people in the household.

Rubric

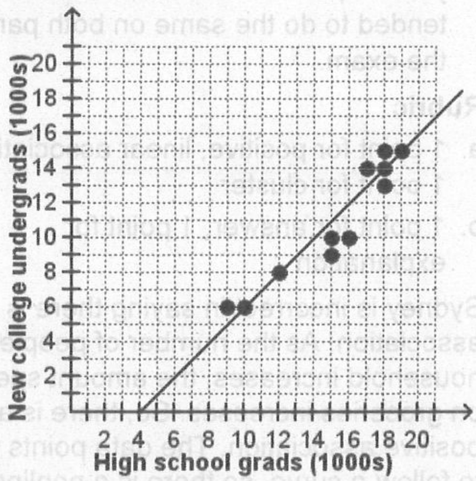
a. 1 point for graph; 1 point for relationship

b. 1 point for line; 1 point for explaining why it's a good fit

c. 1 point

5. Walter's claim is not correct. The data from the table show a linear association. A trend line can be drawn as shown below. It is a good fit for the data because all of the points are close to the line and there is about an equal number of points above and below the line. The trend line appears to pass through (13, 9), so there are approximately 9000 new college undergraduates if there are 13,000 high school graduates.

Possible trend line:



Rubric

1 point for answer; 1 point for graph; 1 point for trend line; 1 point for explanation; 1 point for predicted value

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8.SP.3 Answers

1. C
2. B
3. B
4. F
5. C
6. A
7. H
8. The trend line passes through the points (0, 0) and (40, 15). Since the slope of the trend line is $\frac{15-0}{40-0}=\frac{15}{40}=\frac{3}{8}$, there are about 3 full-page ads for every 8 pages of the magazine. So, about $\frac{3}{8}=0.375=37.5\%$ of the pages in the magazine are full-page ads.

   Rubric

   1 point for answer; 1 point for explanation

9. a. The trend line passes through the points (0, 80) and (5, 20). The slope of the trend line is $\frac{20-80}{5-0}=\frac{-60}{5}=-12$, and the a-intercept is 80. So, the equation of the trend line is $a=-12t+80$.

   b. The constant 80 represents the age of someone who on average goes to a movie theater 0 times per month. The constant −12 represents the decrease in age for each increase of 1 in the average number of times a person goes to a movie theater per month.

   c. A person who on average goes to a movie theater 3 times per month is predicted to be 44 years old.

   $$a=-12(3)+80$$
   $$=-36+80$$
   $$=44$$

   Rubric

   a. 1 point for answer;
   1 point for showing work

   b. 1 point for each interpretation

   c. 1 point for answer;
   1 point for showing work

10. Since the trend line passes through the points (0, 5) and (100, 35), the slope of the trend line is $\frac{35-5}{100-0}=\frac{30}{100}=0.3$, and the p-intercept is 5. So, the equation of the trend line is $p=0.3w+5$.

    If the team wins 0 games during the season, the number of people who attend the last game is equal to the p-intercept. So, the number of people who attend is predicted to be 500. So, Morgan's first claim is incorrect.

    The additional number of people who attend the last game for every game won is equal to 100 times the slope. So, for every game won, an additional $0.3 \cdot 100=30$ people are predicted to attend the last game. So, Morgan's second claim is also incorrect.

    Rubric

    1 point for answer; 1 point for each correct value; 2 points for explanation

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8.SP.4 Answers

1. D
2. C
3. B
4. A, D, E, F
5. Find the relative frequencies of computer preference based on age group.

	Laptop	Desktop	Total
40 years old or older	$\frac{38}{93} \approx 0.409$	$\frac{55}{93} \approx 0.591$	1
Under 40 years old	$\frac{86}{107} \approx 0.804$	$\frac{21}{107} \approx 0.196$	1

Notice that the computer preferences for those 40 years old or older are not the same as the computer preferences for those under 40 years old, so there is an association between age and computer preference. Computer users 40 years old or older are more likely to prefer using a desktop computer, whereas computer users under 40 years old are more likely to prefer using a laptop computer.

Rubric

1 point for description; 2 points for explanation

6. The relative frequency of people who prefer summer among all people polled is $\frac{63}{100} = 0.63 = 63\%$. The relative frequency of people who prefer summer among those who prefer fall is $\frac{27}{52} \approx 0.519 = 51.9\%$. Since 63% of all people polled prefer summer while only 51.9% of the people who prefer fall also prefer summer, there is an association between a preference for summer and a preference for fall: Of the people polled, those who prefer fall are less likely than people in general to prefer summer.

Note that a similar argument can be made using the relative frequency of people who prefer fall among all people polled, $\frac{52}{100} = 0.52 = 52\%$, and the relative frequency of people who prefer fall among those who prefer summer, $\frac{27}{63} \approx 0.429 = 42.9\%$. Since $42.9\% < 52\%$, those who prefer summer are less likely than people in general to prefer fall.

Rubric

1 point for description;
2 points for explanation

7. a.

	Dem.	Rep.	Ind.	Total
Male	24	24	32	80
Female	36	36	48	120
Total	60	60	80	200

b. Find the relative frequencies of political party based on gender.

	Dem.	Rep.	Ind.	Total
Male	$\frac{24}{80} = 0.3$	$\frac{24}{80} = 0.3$	$\frac{32}{80} = 0.4$	1
Female	$\frac{36}{120} = 0.3$	$\frac{36}{120} = 0.3$	$\frac{48}{120} = 0.4$	1

Since the relative frequencies for each political affiliation are the same for both genders, there is no association between gender and political affiliation. Males and females are equally likely to be members of each political affiliation.

Rubric

a. 2 points
b. 1 point for answer; 1 point for explanation

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8. Edwin found the correct relative frequency of students who prefer nonfiction books, but he did not find the correct relative frequency of students who prefer nonfiction books among those who prefer math classes. He found the ratio of students who prefer nonfiction books and math classes to the number of students polled. Instead, he should have found the ratio of students who prefer nonfiction books and math classes to the students who prefer math classes.

   The correct relative frequency of this group is $\frac{49}{86} \approx 0.57 = 57\%$. So, students who prefer math classes are more likely than students in general to prefer nonfiction books.

   Rubric
   2 points for identifying the error;
   2 points for correct association

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